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BREXIT: The impacts on the Sovereign Bond Yields

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After almost one year to decide which Master course I should chose, I have chosen the right one. This master degree gave me back the passion for studying and learning. Even though currently I am not applying specific concepts in my professional functions, I felt that this course provided me more knowledge and technical skills and sensitivity in terms of mathematics and risk management.

Luck is always needed, and I found a great student group, Mariana Coelho, Inês Claudina and João Ildefonso, with whom I shared my difficulties, learned and also shared my experience and knowledge. Studying and working at the same time it is surely not an easy task. Together we supported and motivated each other and turn this journey easier and enjoyable.

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This journey indeed represented to me an endurance test, despite the all complexity of the studied subjects along the master course and, simultaneously, managing and balancing with my professional and personal life, turning the last two years and a half into one of the biggest challenge of my life.

Resumo

O Brexit tem provocado alguma incerteza nos mercados, nomeadamente no europeu. Muitos estudos, estimativas e especulações foram feitas sobre os impactos económicos, no investimento, em termos sociais e até políticos.

No âmbito da minha atividade profissional numa instituição financeira (até setembro de 2019) tive a responsabilidade de acompanhar as dívidas soberanas numa ótica de apetência pelo risco bem como a concentração a este tipo de ativos, sabendo que este tema era também uma preocupação do Banco Central Europeu.

O objetivo deste trabalho foi compreender melhor o evento Brexit e o impacto sobre as *yields* das dívidas soberanas europeias mais importantes com maturidade a 10 anos, nomeadamente, a Alemanha, Holanda, Reino Unido, Irlanda e Dinamarca. Os critérios de seleção foram a proximidade geográfica, importância na UE e alguns dos países periféricos que estiveram sujeitos a um plano de resgate recentemente (após a crise de 2012).

Para isso foram utilizadas as *yields* trimestrais históricas bem como variáveis económico-financeiras (dívida em % do PIB, taxas de desemprego e taxas interbancárias de referência), com objetivo de desenhar modelos que fossem capazes de reproduzir, de forma mais aproximada possível, a realidade e serem utilizados para *forecasting*.

As variáveis foram analisadas no que respeita às correlações estática e dinâmica, estacionaridade, causalidades de Granger e cointegrações (por via do VECM) entre si, testes que foram realizados com o objetivo de selecionar as variáveis para um modelo.

As conclusões foram que efetivamente existe uma correlação estática e de longo prazo entre algumas destas soberanias. Foram criados dois modelos, ambos estatisticamente significativos tendo incidido sobre duas principais soberanias: a alemã (que habitualmente é utilizada como *benchmark* da taxa de juro sem risco) e a do Reino Unido, naturalmente por ser a soberania sobre foco da discussão deste tema do Brexit. Não obstante, e pela informação analisada, reconhece-se que outras relações poderiam ser modelizadas.

No final da dissertação, os dois modelos foram utilizados para exercício de *forecasting* e foram realizadas outras simulações com dados mais recentes, de modo a perceber como as *yields* alemã e inglesa se comportariam.

Foi equacionada a inclusão neste trabalho outras dívidas soberanas (portuguesa, grega, espanhola e italiana) e outras maturidades (a 5 anos e 3 meses). Contudo, devido à dimensão limite da dissertação, foi decidido não incluir. No entanto, foi possível constatar que algumas *yields* possuíam comportamento semelhante ao que a dívida a 10 anos demonstrou pelo que acreditamos que seria matéria merecedora de uma análise similar à que foi realizada neste trabalho.

Palavras-chave: Correlação estática, Correlação dinâmica, Causalidade de Granger, Cointegração, Yields.

Abstract

Brexit has been causing uncertainty in the markets, particularly within Europe. Many studies, estimates and speculations have been made regarding the economic, investment, social and even political impacts. Within the scope of my professional activity in a financial institution (until September 2019) I had the responsibility to monitor sovereign debts from the perspective of risk appetite as well as the concentration on this type of assets, knowing that this topic was also a European Central Bank concern.

The purpose of this work was to understand the Brexit and the potential impacts on the most important European sovereign yield debts of the 10 year maturity, namely, Germany, Netherland, United Kingdom, Ireland and Denmark. The selection criteria were geographically proximity, importance within EU and some of the periphery countries that were subject to a bailout plan recently (after the 2012 crisis).

For this study, historical quarterly yields were used as well as economic-financial variables (debt in % of GDP, unemployment rates and interbank reference rates) from January 2000 to March 2019 in order to design models that could reproduce, as closely as possible to the reality and to use for forecasting.

These variables were analysed in terms of static and dynamic correlations, stationarity, Granger causalities and cointegration (via VECM) with each other, tests that were performed in order to select the variables for a model.

After performing the mentioned test, it was found static and long-term correlations. Two models were created, both statistically significant, being the dependent variable (the one which is explained) the two main sovereign yields: the German bond yield (which is usually used as a reference/benchmark for the risk-free risk rate) and the English one (since it is the most important role player in this event - Brexit). Notwithstanding, and according with the data analysis, we recognize that other relations could be modelled.

In the end of the dissertation, both models were used for forecasting exercises and were conducted other simulations with more recent data, in order to understand how the German and English sovereign bond yields respond.

It was also evaluated the possibility to include other SBYs (Portuguese, Greek, Spanish and Italian) and maturities (for 5 years and 3 months) in this work. However, due to the length limit, it was decided not to include them. However, it was possible to conclude that some of these yields showed a similar behaviour comparing with the SBY for 10 years, wherefore we believe that this data would be worth to be analysed as performed in this work.

Keywords: Static correlation, Dynamic correlation, Granger Causality, Cointegration, Yields.

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List of Abbreviations

EU – European Union
ADF – Augmented Dickey-Fuller
AIC - Akaike Information Criterion
AR – Autoregressive model
ARCH - Autoregressive Conditionally Heteroskedastic
BOE - Bank of England
BG – Breusch-Godfrey
BPG – Breusch-Pagan-Godfrey
CIP – Confederação Empresarial Portuguesa
CDSp – Credit Default Spread
CDSw – Credit Default Swap
DW – Durbin-Watson
ECB – European Central Bank
ECSC – European Coal and Steel Community
ESM - European State-Members
ESMA – European Securities and Markets Authority
EY-AM&A – Ernst & Young – Augusto Mateus & Associados
GDP – Gross Domestic Product
HUR – Harmonized Unemployment Rate
KPSS – Kwiatkowski-Phillips-Schmidt-Shin
LM – Lagrange Multiplier
MA - Moving Average Model
OLS – Ordinary Least Squares
PP - Phillips-Perron
RMSE - Root Mean Square Error
SBY - Sovereign Bond Yield
TB – Treasury Bond
UK – United Kingdom
USA – United States of America
VAR – Vector Autoregressive model
VECM – Vector Error Correction Model

1 Introduction

The public debt of European State Members (ESM) as percentage of GDP and the high dependency of external source funding have been a concern within the European Union (EU) authorities and State-Members. The European crises, particularly in 2010, led to the high rates of Sovereign Debt securities in some particular ESM causing the increase of respective financial costs. In this crises it was witnessed the contagious effects of the peripheral ESM on other ones.

On the other hand, private investors as well as Financial Institutions (mainly banks) have been looking for sovereign debt securities in order to increase the collateral to easily access to the European Central Bank (ECB) funds, diversify portfolios with low capital consumptions for the banks (and low risk weighted assets) as well as to reduce the costs of the liquidity excess. Moreover, it offers some return (depending on the ESMs sovereign debt) at a low risk and low costs when compared to other investments (for example equity stocks, private loans, private funds, derivatives).

The Brexit has caused additional concerns related to the political, social, economic and financial stability in Europe.

Since the creation of the first European community organism (Benelux and European Coal and Steel Community, ECSC, in 1952), Europe has never seen one of its State-members leaving the EU. Many treaties and other accords took place in Europe since then in order to include and integrate more State-members (in 2012 occurred the last inclusion process of a new ESM, Croatia) and simultaneously to promote prosperity, safety and cohesion among the ESMs, as well as to standardize policies concerning the economically and safety matters, respecting for the human rights, creating a common market and free movement of people. Since the beginning, all of this have been the main goals of EU and the main drivers of its policies.

Brexit symbolizes the first significant event related with the decrease of ESMs. It all started on January 2013 when David Cameron refers the possibility of making a referendum in order to discuss the role of UK in the EU. He used the referendum as part of its election program which was specifically regarding the leave of UK from EU. In May 2015, David Cameron won the elections. The Brexit negotiation process started by the referendum which has occurred in 23rd June 2016 in UK and the leaving scheduled (for the first time) on 29th March 2019.

These were some of the quotes from politics, analysts and other public figures[1] along the last years, and, as can be seen, there have been no uniform opinion regarding the Brexit and its consequences.

"*Of course Brexit means that something is wrong in Europe. But Brexit means also that something was wrong in Britain.*" - Jean-Claude Juncker (President of the European Commission from 2014 to 2019)

"*Labour are a danger to our security and our economy and are wholly incapable of negotiating the best Brexit deal for Britain."* - Amber Rudd (British politician and Secretary of State for Work and Pensions from 2018 to 2019)

"So we are getting ready to come out on October the 31st. Come what may . . . Do or die. Come what may." - Boris Johnson (British politician and Leader of the Conservative Party) since July 2019

"*Brexit is the other face of the refugee crisis - tensions that lead to stasis, external risks that lead to asymmetric shocks.*" - Emmanuel Macron (President of France since May.2017)

"Brexit is a disaster, Italy won't be real about its debt, and the European Union is in trouble." - John Layfield (financial commentator featured regularly on Fox News Channel)

"We will do all this in the knowledge that with the departure of Great Britain, a potential competitor will of course emerge for us. That is to say, in addition to China and the United States of America, there will be Great Britain as well" - Angela Merkel (German politician and Chancellor of Germany since 2005)

[1] See more Quotes in https://www.brainyquote.com/topics/european-union-quotes

"*Brexit is the tip of the iceberg - there are so many endlessly complex permutations beneath the surface that nobody will see them all until Britain leaves the EU and maybe not even then."* - Stewart Stafford (American movie producer)

"Brexit is the best thing to happen for Russia, for America, for Germany, and for democracy." - Nigel Farage (British politician and leader of the Brexit Party since 2019)

"The most difficult part of Brexit will be to figure out the trade regime between the U.K. and the rest of the E.U. because the level of trade integration between the members of the E.U. is the deepest in the world and integrates regulations that govern how products and services are produced and sold within the E.U." - Arancha Gonzalez (Spanish lawyer and Assistant Secretary - General of the United Nations)

"The British have chosen liberty with Brexit and can congratulate themselves every day." - Marine Le Pen (French politician and member of the National Assembly)

There were many reasons that may have trigger the referendum. Some authors and journalist believed that this matter should have been discussed and handled by the UKs government authorities instead of submitted this decision to the opinion of UKs people. Moreover, it is also believed that the voters should have had more information about the pros and cons of leaving EU in order to decide wisely.

It is difficult to choose the main reason which may have triggered the referendum. Notwithstanding, it is possible to refer some triggers (the major part of them have the consensus among the authors) that led to the referendum in June 2016:

- Crisis in Europe (unemployment, high sovereign debt levels in the periphery countries, stagnation), mainly the one occurred in 2010;
- The mass migration of people (caused by war in the middle east and Syria);
- Terrorism: many attacks occurred in Belgium (2016), Germany and France (in 2015, 2016 and 2017). Other events also occurred in 2017;
- The media, the academic and journalists of Brexit supporters as well as the euro-sceptical and nationalists political and social forces;
- Political campaign of David Cameron;
- The election of Donald Trump in United States of America.

The Brexit also led to concerns regarding the possible contagion and spill over effects to other European countries and respective negatives impacts:

- The elections in France (ended in May 2017 with the election of Emmanuel Macron), Germany (March 2017, being elected Angela Merkel) and in Austria (in October 2017 was elected Sebastien Kurz). Although nationalists and conservative forces have not won they gained more importance in the turnouts;
- Nordic European countries as well as some eastern ones (Czech Republic) which have been more euro-sceptical;
- The referendum to decide the independence of the region of Catalunha (in October 2017). The referendum was considered illegal by the Spanish Government;
- The impacts on the economic and financial stability in Portugal, Greece, Italy and also in the UK's neighbour countries (namely Ireland, Netherlands, Belgium and Denmark).

After the Brexit referendum, the uncertain about the future of EU became more evident among the economists, traders and investors. Many questions regarding the Brexit negotiations have being made since the referendum, namely concerning the shock on EU budget (estimated of 9 billion euros[2]), commercial accords, Schengen Agreement, the single market and in the agriculture and fisheries policies.

[2] Jacobs, Francis B. 2018. The EU After Brexit – Institutional and Policy Implications, page 105

It is also believed that EU would enlarged in the future. This means that EU may integrate more European countries. Since part of the non-ESMs are considered more conservative and poor countries a new integration process could turn EU economically and financially weaker. Concerning the institutional and social repercussions it can be highlighted the impacts in the European Parliament and Commission organisms, affecting jobs, communication (possibly the official language) and a new mass migration of people from and to UK.

The Brexit negotiation process has led to several forecasts regarding the future of EU. Many questions were being asked:

- Which party, UK or EU, will be more affected after this change?
- Or, on the other hand, there will be some competitive advantage, caused by decisions which are not compliant with the European policies, granted to the UK?
- How do the neighbour countries (highlighting, Ireland, Denmark, Netherland) will handle after the disintegration of EU?
- Is it possible to avoid the budget problems?
- How do the rest of State-members of EU will negotiate with UK and what will be the effects on the European market?

Under the uncertain context, financial markets, investors, financial institutions as well as the sovereign's organisms have shown some reluctance about all this process, although the European Commission have been trying to mitigate some uncertainty by showing possible scenarios regarding the future of EU.

The White paper on the future on Europe, published by the Europe Commission, at 1st of March 2017, defined five different scenarios for Europe 27 by 2025, all of them with impacts on the single market and trade matters, on the economic and monetary union, on migration and security matters, foreign policies and defence, EU budget, and on the capacity to deliver.

- Scenario 1: it forecasts the maintenance of the EU as it is, focusing on employment, investments, strengthening the single market and addressing the challenges concerning the environment and the sustainable growth.
- Scenario 2: it is highlighted the single market as the main strategic driver of EU, without a compulsory cooperation in security and migration matters. Apart from the risk of integrity of the single currency and, eventually, a new crisis, there would be struggles regarding the deals between EU countries.
- Scenario 3: in this scenario there would be more bilateral or multilateral accords regarding some specific matters (defence and security, budgetary). This would provoke less transparency related to the deals, turning the economic and financial markets more complex.
- Scenario 4: the main goal of EU in this context would be to increase efficiency and cooperate in very specific matters, like in R&D, security and management of boarders. Other matters would be sacrificed like public health, employment and social policies.
- Scenario 5: On the opposite of a less cooperation as forecasted in scenario 4, the EU would become even closer, united in a full cooperation across all policy areas (migration, investments, economic, markets, energy and innovation). Defence and security matters would be priorities, as well as the fight against the global warming. State-Members would loss some individual power driven by the common policies and full integration.

In 14th December of 2017 the Joint Declaration on the EUs legislative priorities for 2018-2019 was settled, a document which represents the seven short-term priorities for EU. Once more, matters such as security, migration, employment and investment, protection of the workers, single digital market, environmental policies and fostering economic growth and democracy (and others, such as the fight of the fiscal evasion and protection from the robotic and artificial intelligence) are still the priorities for 2018 and 2019.

On the other hand, after internal struggling on the UK's Parliament regarding the leaving accord, UK defined three possible scenarios in December 2017 (instead of 5 as defined by the EU). The three scenarios were[3]:

- Scenario 1: "Hard Brexit" means leaving without accord, which presume the existence of transactions fees and barriers. According to the National Institute of Economic and Social Research economists it is expected a GDP contraction of -5.5% by 2030 and the Bank of England foresees -3% in a short-term. This scenario will be more adverse to UK if no third party agreements prevail;
- Scenario 2: Art.º 50 (procedures concerning the leaving of the EU) extension, which must be authorised beforehand by the UK and EU in their respective responsible bodies;
- Scenario 3: Brexit with accord. It was foreseen by UK's economists that, despite the agreement, a negative impact would occur although not so severe as the "Hard Brexit" (-1.75% to -3.9% on the GDP).

The EU have been committed regarding the security matters, growth and prosperity, all this in a difficult context related to the migration and other external events which may affect the balance of each State-members and the integrity of EU (climate, high technology, conservative political forces, euro-sceptics, and other world powers, like USA, China and Russia). At the same time the Brexit negotiation may will cause more concerns about the EU politic and economic stability if it does not defend its position and interests in benefit from UK.

Other studies and papers regarding the financial markets and sovereign debt were performed along 2017 – 2019 period.

Bloomberg published some news regarding the Brexit negotiations and its consequences on financial markets. In 5th of February 2019 they informed that EU and UK did additional efforts in order to maintain trillions of euros in transactions. They also said that Bank of England reached an agreement with ESMA to ensure derivatives would continue to be managed by London Stock Exchange Group Plc., Intercontinental Exchange Inc. and London Metal Exchange clearinghouses.

In the report Brexit – As consequências para a economia e as empresas portuguesas (CIP – Confederação Empresarial de Portugal and Ernst & Young – Augusto Mateus e Associados, September 2018, pages 86 to 91) were summarized the main conclusions of seventeen studies, of which twelve clearly estimated a reduction of GDP (investment, exportations) across the Europe (particularly in the UK) and other adverse repercussions. Only one estimated a positive outlook (the rest are not much straightforward regarding the positive or negative impact, for example, in impacts on fees, trade accords and employment). Considering the UKs imports, CIP highlights Portugal as the fourth destination of Portuguese exportations of goods and the first of services. This means that even weaker ESMs would not escape from the potential consequences of Brexit.

One of the twelve studies - The immediate economic impact of leaving the EU (HM – Treasury analysis, May 2016, pages 18 to 19) - approach the impact in the sovereign debt, namely by the volatility of the Sovereign CDSw and in the index of CDSp. The evolution showed an increase in the UKs protection costs in November 2015 – May 2016 period. It should be highlighted that it also showed, in the same period, the same evolution in the German and French CDS. Additionally, this study referred that the longer term sovereign debt (10-year bond) would also rise (more risk and uncertain means more return) as well as the stock of sovereign debt (pages 41, 51, 53, 56), and there would be a very possible

[3] Caixagest - Técnicas de Gestão de Fundos, S.A. (Grupo Caixa Geral de Depósitos), 2019. Brexit – Perguntas & Respostas, 2019, Comentários de Mercado.

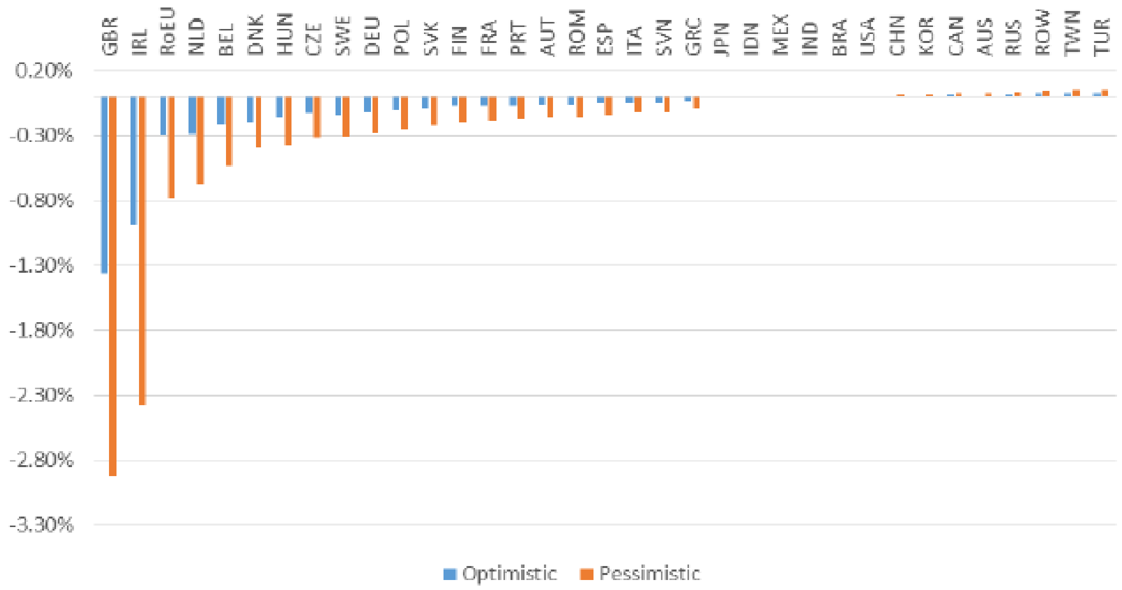
contagion to other ESM sovereign debt (page 43) (according to the historical relationships equity risk premium of EU countries are related to the UK's).

Other paper from HM – Treasury Analysis, "Economic impact of EU membership", have also concluded that Brexit will possible lead to negative impacts in the UKs export destinations, highlighting Switzerland, Netherlands, Germany and Ireland. It is also said that this would provoke potential adverse impacts to other ESMs.

Another paper, Brexit 2016 – Policy analysis from Centre for Economic Performance (June 2016) have clearly presented that all EU countries will suffer with an income decrease in UK of 26 billion pounds, while the rest of EU could lose 12 billion pounds. In the overall the EU countries may easily decrease its GDP -0,12% to -0,29% (pages 16 to 17).

From the same institute Centre for Economic Performance another paper focus on the living standards, "The consequences of Brexit for UK trade and living standards" (Swati Dhingra, et. al. 2016). As can be seen in Figure 1.1, England, Ireland, and Netherland will represent the top three countries with the higher negative impacts in terms of living standards.

Figure 1: The effect of Brexit on living standards across countries



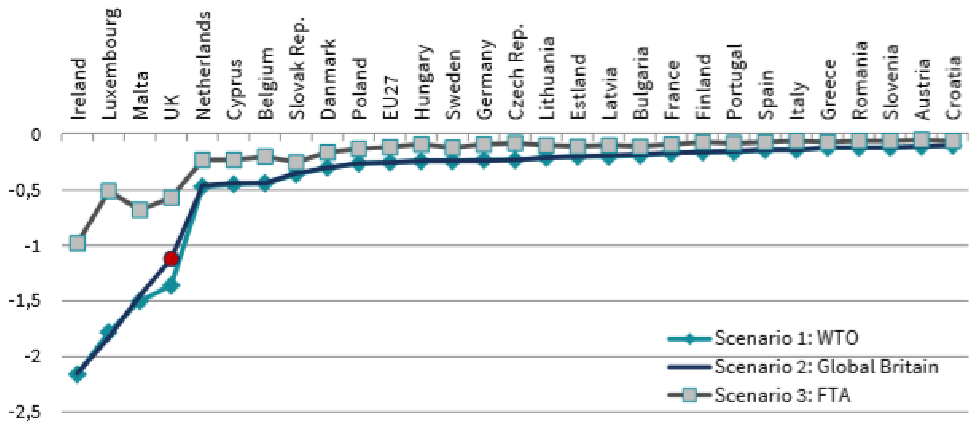
Source: CEP calculations (see Dhingra et al, 2016, for technical details).
Notes: Same assumptions as in notes to Table 1 except net fiscal savings not included (since we do not know how Brexit would affect the budget contributions of other EU members).

Figure 1.1 - The effect of Brexit on living standards across countries

A more recent paper, Brexit – History, Reasoning and Perspectives (Trotiño, David and Chochia, Tanel, 2018, page 106) also refer that "most of the analytical reports on the effects of Britain leaving the EU were rather pessimistic (and, in some cases, apocalyptic), suggesting negative outcomes for the household incomes, economic growth, gross domestic product or trade".

Another interesting paper was published on November 2017 by EconPol Europe – "Economic Effects of Brexit on the European Economy" (Gabriel Felbermayr, et al., 2017). This paper displayed a chart (Figure 1.2) with the potential impacts on Real per Capita GDP (in %) in the main EU countries, based on three different scenarios[4].

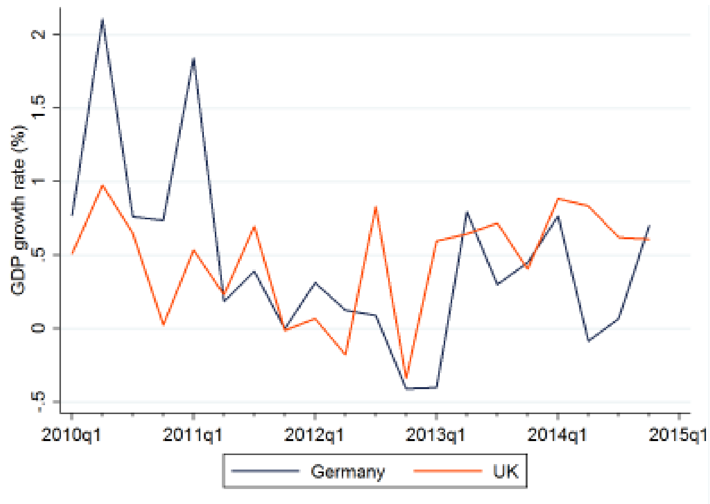
[4] For more detail see Felbermayr, Gabriel, 2017. "Economic Effects of Brexit on the European Economy", pages 16 to 17.



Source: Felbermayr et al., 2017c. Own representation. Relative to the status quo in 2014. These effects do not include net transfers to the EU.

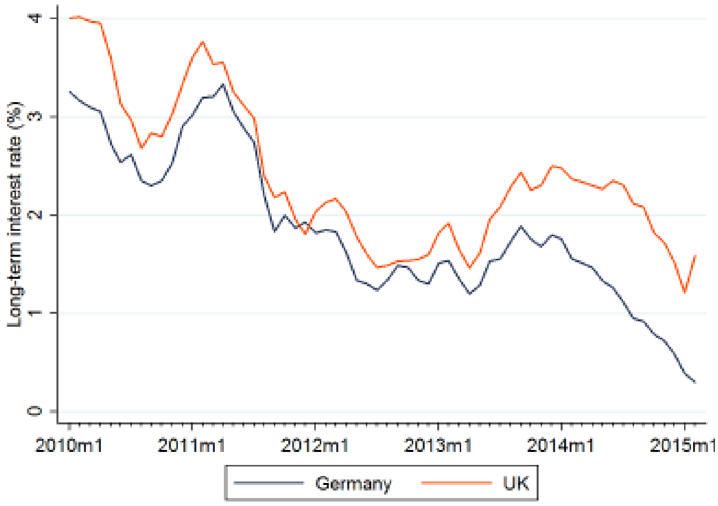
Figure 1.2 - Potential impacts on GDP

The same author compared, using six charts, specifically the German and the English GDP, economic trends, interest rates for 10 years bonds, inflation and government debt along the 2010Q1 – 2014Q4 period (Felbermyr Gabriel and Rahel Rachel 2015, “Costs and benefits of a United Kingdom exit from the European Union”, pages 13 to 17). The charts suggested a correlation between the two countries in all those variables, although at a different pace. See bellow some of these charts:



Source: Data from EZB, Quarterly GDP data, seasonally and working day adjusted. Own representation.

Figure 1.3 - Economic trends in Germany and the UK, Q1 2010-Q4 2014



Source: Data from EZB.

Figure 1.4 - Long-term interest rates for 10-year bonds, 2010-2015

In a nutshell, the uncertainty does not come from the question if there will be any adverse consequences. The question is about the magnitude of all this impacts and the remediation actions that must take place to mitigate them.

The Sovereign Debt crises in 2010 (partially caused by the financial crisis 2008-2009) triggered the necessity of developing several studies in order to understand the source of it as well as to study these events concerning the contagion between ESM sovereign debts. Many authors, although using different methodologies or approaches, detected contagious effects, highlighting higher sensitivity of the periphery ESM (Greece, Portugal, Spain and Italy) than central ESM.

There is not a universal definition of contagion in this context although Pericoli and Sbracia (2003) provide five different types:

- The probability of a crisis in one country rises sharply as a response to a crisis in another country;
- The increase in asset price volatility is cross-national;
- Co-movements of asset prices are not fundamentally driven;
- The co-movements of financial assets between countries increase significantly;
- The transmission mechanism between countries changes conditional on a crisis in one of the countries, also leading to a change in the co-movement of those countries' asset prices.

In the paper "Financial Contagion and the European Debt Crisis" (Missio and Watzka, 2011, pages 22 to 24) was argued that Greece sovereign debt problems triggered the European sovereign crises. The authors performed a Dynamic Conditional Correlation Model to analyse the correlation structure of Greek, Portuguese, Spanish, Italian, Dutch, Belgian and Austrian bond yield spreads over the German one and this study suggested the existence of Greek contagion on Portuguese, Spanish, Italian and Belgian yields.

In "The pricing of sovereign risk and contagion during the European sovereign debt crisis" (John Beirne and Marcel Freitzscher, 2013, page 81) paper, distinguished three types of contagion during 2008-2012 period: regional, herding (caused by temporary overreactions), and fundamentals (resulted by reactions, which were taken based on the country's economic fundamentals).

In the paper "Deciphering financial contagion in the euro area during the crisis" (Tola and Wälti, 2014, pages 9 to 10), performed based on Favero and Giavazzi (2002) work, suggests the existence of contagion due to specific/individual country's shocks. This idea (empirical approach) underlies that sovereign yield depends on its shocks as well as other country's shocks (including lags). These authors also argued that this one should be isolated from the global and euro shocks since ESM can be affected by a global shock.

This dissertation aims to understand the impact of the Brexit in a specific set of ESMs sovereign debt securities yields by using historical data of SBYs (for Denmark, France, Germany, Ireland, Netherland and United Kingdom) for 10 years maturity (the most typical debt securities maturity), macroeconomic (as sovereign debt as percentage of Gross Domestic Product, Harmonised Unemployment rate - HUR) and other financial data (as the UK and EU refinancing rates). This data will be studied with a statistical/econometrical models, being the purpose try to find and explain correlation, causality and other properties by relating them to each other. In order to attain these, we analyse regression models and perform forecasting.

The core of this thesis is based on four chapters. The first one, the "Review of Technical concepts", on which is explained the underlying theory of the statistical models used in this study. The second chapter, named "Data sets and empirical analysis", has all the variables description, used in this study, an empirical overview of sovereign debt in the time-line defined, and all the statistical tests (correlation, causality, stationarity, cointegration, vectors and regressions). After the variables analysis and regressions designing, in the fourth chapter can be found the forecasting exercises using those regressions, in order to check the forecasting ability. The final conclusions can be found in the chapter 5 (notwithstanding, other conclusions are pointed out along this work).

It was used the E-views software for the statistical tests and the respective outputs.

2 Review of Technical concepts

The Brexit process may have multiple outcomes, since many variables should be take into account. Economic, political, social, financial, certainly, are fields that will be impacted by this event. Moreover, and considering the turbulence across all the world (for instance, USA and China commercial war) more constraints will take place in order to predict and easy measuring all the impacts.

Correlations can be studied using a static or dynamic approach. This will be important for the modelling process since we will look for variables which are related somehow and use them to explain SBYs.

For a static correlation approach it will be used the Pearson's correlation coefficient[5]. This ratio represents the degree of linear relationship between two variables and is computed by dividing covariance between those variables and the square root of the variances product, generically:

$$R = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2} \times \sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

Equation 2.1 - Pearson's coefficient - Correlation

As closer to 1 or -1 (boundaries), the correlation is strongly positive or negative, respectively. The covariance measures the behaviour of those variables, namely if they have the same variations and how is the dispersion level of them. The variance is computed by doing the square of standard deviation. The standard deviation measures individually the dispersion, comparing the values to its average (which can be negative).

This represents a static measure since this parameter does not consider the time effect along the series.

Since this study also focus on correlation in order to use all variables behaviour to reproduce other one is also important to measure correlation in a dynamic timeline, as one variable can be impacted by the past behaviour of another one.

The Causality Granger[5] method use past values of a specific variable (exogenous variables) to explain another one today (endogenous variables). This kind of relationship is called unidirectional Granger Causality. If two variables have Granger causality on each other we are in the presence of bi-directional Granger Causality.

The dynamic regression equation can be written by:

$$Y_t = \beta_j \times X_{j,t-n}$$

Equation 2.2 - Dynamic regression equation for Granger Causality

where Y is the explained variable at the present time moment t, β is the coefficient matrix of the exogenous variable (for j variables) for which it takes the $n - lags$ (in other words, past values). This is a statistical method which test the following hypothesis:

$$H_o: All\ \beta_{j,t-n}\ coefficients\ for\ variable\ x_j\ are\ null$$

$$H_1: at\ least\ one\ \beta_{j,t-n}\ is\ not\ null$$

The null hypothesis it is equivalent to say that there is no Granger causality between X and Y variables, in the sense that X do not Granger cause Y. The obtained $p - values$ are compared with the significance level α defined in the analysis. The confidence level is:

[5] See Brooks, 2008, Introductory Econometrics for Finance, second edition.

$$1 - significance\ level\ (\alpha) = Confidence\ level$$

Equation 2.3 - Confidence level

Depending on the $p - value$ and as much closer to 0 means that there is a high confidence level of rejecting the hypothesis. In this case, if the $p - value < significance\ level$ it is rejected the H_o, meaning that past values of variable X explain Y at the present moment t and in consequence Granger causality exist for that significance level.

It is also important to guarantee that the time series follows a stationary process (which means that the process do not possess a unit root). Otherwise, in the presence of a unit root, we can compromise all the statistical results, obtain a biased analysis and false forecasting figures.

The variables used in this study are not "well behaved", since they are subject to continuous fluctuations in trend and variance. For this reason, volatility may be also not constant along the series. Concerning the stationarity it will be performed three tests - Augmented Dickey-Fuller (ADF)[6], Kwiatkowski-Phillips-Schmidt-Shin (KPSS)[6] and Phillips-Perron (PP)[6] and the purpose is to verify if the series are stationary, or, in other words, has a constant mean, variance and covariance:

$$E(y_t) = \mu$$

Equation 2.4 - Constant mean

$$E(y_t - \mu)(y_t - \mu) = \sigma^2 < \infty$$

Equation 2.5 - Variance

$$E(y_{t1} - \mu)(y_{t2} - \mu) = \gamma_{t2-t1} < \infty, \qquad \forall\, t_1, t_2$$

Equation 2.6 - Covariance

There are some techniques which allows to stabilize the series in order to smooth them and get better results (differences, logarithmic transformation). If we can attain stationarity after the application of the first difference operator, as the following equation shows,

$$\Delta Y_t = Y_n - Y_{n-1}$$

Equation 2.7 - First difference operator

then, we say that the time series it is integrated of order 1 ($I(1)$). The reason for performing three tests for the stationarity is to prevent wrong conclusions. We have the following hypothesis:

Augmented Dickey-Fuller (ADF)	H_o: $existence\ of\ unit\ root$ H_1: $stationarity$
Kwiatkowski-Phillips-Schmidt-Shin (KPSS)	H_o: $stationarity$ H_1: $no - stationarity$
Phillips-Perron (PP)	H_o: $existence\ of\ unit\ root$ H_1: $stationarity$

Figure 2.1 - Hypothesis for Unit roots and Stationarity

The hypothesis are rejected or not rejected depending on the $p - values$ or the test statistics. For the ADF and PP tests we compare the test statistics with the critical values, rejecting the null when the test statistics are greater than the critical value (at a certain significance level). This means that the series is not stationary. If the $p - value$ is higher than the significance level, then we conclude that it has a unit root (non-stationary).

[6] More details related to unit root and stationary tests, see Brooks, 2008, Introductory Econometrics for Finance, second edition.

As for the KPSS test the output is measured according with the LM-statistic values, and it should be read as the following

Significance level	0.10	0.05	0.01
LM-test critical values	0.119	0.146	0.216

Table 2.1 - LM-test critical value

If the LM-statistic is higher than the critical values than the null hypothesis is rejected, meaning that is non-stationary.

Since all SBY are integrated of order one, $I(1)$, it is possible to study the cointegration between these variables by using the Johansen methodology. This method will be used instead of the Engle-Granger one since we are looking for more than one cointegrating vector.

If exists, the cointegration relation between non-stationary time series allows to establish long-run relationship between variables, or, in other words, allows to understand if linear combinations are possible to be made in order to found convergences to a long-term relationship balance.

The hypothesis for the Johansen cointegration tests are the following:

$$\boxed{\begin{array}{c} H_o: no\ cointegration \\ H_1: H_o\ is\ not\ true \end{array}}$$

Table 2.2 - Hypothesis for the Johansen cointegration tests

The decision criteria are based on Trace and Maximum Eigenvalue statistics. Both should be above the critical value of 5%[8].

Another approach considered in this work was to apply the Vector Autoregressive Model (VAR) that used all variables as endogenous. The base of this econometric methodology is that every variable is explained by its own historical values ($n-\ lags$). In presence of more than one cointegration vector and $I(1)$ variables we should use a Vector Error Correction Models (VECM), instead of VAR, in order to analyse the short-run and the long-run behaviour of the nonstationary time series. The equation is (and ignoring trend):

$$\Delta Y_t = \prod Y_{t-i} + \sum_{i=1}^{p-1} \Gamma_i \Delta Y_{t-i} + u_t$$

Equation 2.8 - VECM equation[7]

where,

$$\prod = \alpha \times \beta'$$

Equation 2.9 - VECM long-term data

represents the long-term data, where α is the matrix of the velocity adjustment parameters, β is the matrix containing the cointegrating vectors and u is the error term represented by a white noise process[8].

Regarding the cointegration equation can be generally written by the following linear combination between β_s coefficients and lagged variables:

$$ECT_{t-i} = Y_{t-i} + \sum_{i=1}^{p-1} \beta_i X_{t-i} + \beta_0$$

Equation 2.10 - Error Correction term

[7] In the convention equation it can have an exogenous variable, namely Error Correction term of the OLS Residuals.
[8] See Brooks, 2008, Introductory Econometrics for Finance, second edition.

The VECM model should be used after performing the Johansen cointegration test, since the VECM is able to analyse also short convergences.

Another model to be employed in our analysis is the multiple linear regression, which can be generally represented by:

$$Y_t = \beta_0 + \beta_1 \times X_{1t} + \beta_2 \times X_{2t} + \cdots + \beta_k \times X_{kt} + \varepsilon_t$$

Equation 2.11 - Multiple linear regression

These betas (β_k) are found/estimated by using the Ordinary Least Squares (OLS) method, being β_0 the intercept with $y - axis$, the X_{kt} are the variables which will try to explain Y_t , β_i are the coefficients of the independent variables X_{kt} and the error ε_t term is a white noise process. The OLS method basically tries to fit or adjust a line (a hyperplane for the general case) to the different observations at a minimal distance possible to them. In other words tries to minimize the sum of the squared residuals ε_t, as the following expression shows:

$$Min \sum_{n=1}^{N} (Y_t - \widehat{Y_t})^2$$

Equation 2.12 - Ordinary Least squares estimation

being, N the number of observations and $\hat{Y}_t$ the values predicted by the model. This can be showed graphically as in the Figure 2.2 below:

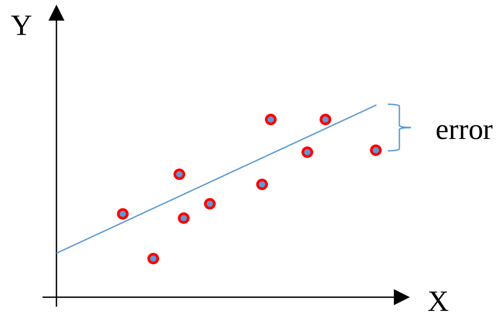


Figure 2.2 - OLS application

The dots are the pairs of observations and the purpose of this model is to find the betas for which the errors will be minimized.

There are some fundamental parameters and assumptions that we have to check in order to conclude if the model has good properties or not. The following ones are the most important ones, and we briefly explain the meaning of them.

Main output indicators/tests meaning:

- The β_s should have $p - values$ below the determined significance level (usually 5% or 10%), so we can rejected the null hypothesis, as described above,

  $$H_o: All\ \beta_{j,t-n}\ coefficients\ for\ variable\ x_j\ are\ null$$

  otherwise, it is not rejected that,

  $$H_1: at\ least\ one\ \beta_{j,t-n}\ is\ not\ null$$

  In other words, we do not reject the model itself;
- R^2 – squared correlation coefficient of the regression. As much closer to 1 greater is the success of predicting the values of the dependent variable within the sample.

- Akaike/Schwarz information criteria: are commonly used as a measure of the model quality.

Residuals assumptions/tests meaning:

- Assumption 1: the expected value of residual is 0.
- Assumption 2: the variance of the residuals is constant:
  - Breusch - Pagan - Godfrey test: null hypothesis is the existing of homoscedasticity.
- Assumption 3: no (auto)correlation between residuals:[9]
  - Breusch - Godfrey LM test: null hypothesis is that there is no serial correlation up to lag order p.
- Assumption 4: the independent variables must be non-stochastic.
- Assumption 5: residuals follow a Gaussian distribution (Normal distribution):
  - Jarque-Bera test with Null hypothesis: normal distribution;

After the model is designed and validated, it should be tested for forecasting performance, that is, we check if the information produced by the model it is far or close to the observed/realised values.

Statistical forecasts can be performed using in-sample and out-of-sample methodology. The difference is related with the use of all observations and forecasting a sample of it (in-sample) instead of design a model with a sample of available information and forecasting another part of the "unknown" available data (out of sample). In this thesis was used both approaches.

Concerning the indicators to verify the model predictability performance, the most common metrics are the Root Mean Square Error (RMSE) and Theil's U-Statistic. The first is given by the following expression,

$$RMSE = \sqrt{\sum_{n=1}^{n} \frac{{e_t}^2}{n}}$$

Equation 2.13 - Root Mean Square Error

where,

$$e_t = y_t - f_t$$

Equation 2.14 - Forecast error

is the forecast error (difference) between the observation and the forecast figures and n the number of forecasts. Theil´s U-statistic is represented by the equation,

$$U = \frac{\sqrt{\frac{1}{n}\sum_{n=1}^{N}(Y_t - f_t)^2}}{\sqrt{\frac{1}{n}\sum_{n=1}^{N}Y_t^2} + \sqrt{\frac{1}{n}\sum_{n=1}^{N}f_t^2}}$$

Equation 2.15 - Theil's U-Statistic

being f_t the forecasted values obtained from a benchmark model (resulted by a random walk). Both should be closed to zero in order to design an appropriate and successful forecasting model[10].

[9] Durbin Watson test parameter: this test derived from the BG LM test and measures if exists first-order serial correlation in the residuals. This test is valid if there is no null intercept, no stochastic variables and no lags in dependent variable.

[10] See Brooks, 2008, Introductory Econometrics for Finance, second edition.

The Theil's U-Statistic forecast error is a result of three parameters: bias proportion, variance proportion and a covariance proportion. The bias proportion is the difference of the means produced by the forecast and the realised data one (also called systematic error) and is represented by:

$$\frac{(\sum_{n=1}^{N} \frac{Y_t}{n} - f_t)^2}{RMSE}$$

Equation 2.16 - Theil's U-Statistic: Bias proportion

being Y_t the observed values and f_t the forecasted values. The variance proportion follows the same idea as the bias proportion. It measures the difference between the variances of the forecasted values and the real ones, being calculated using the following expression:

$$\frac{(\sigma_t - \bar{\sigma}_t)^2}{RMSE}$$

Equation 2.17 - Theil's U-Statistic: Variance proportion

The $\bar{\sigma}_t$ is the variance of the forecasted model and σ_t the real variance. Finally, the covariance proportion represents all other errors (unsystematic error) between the forecasted values and the actual data. It is calculated using the expression below,

$$\frac{2 * (1 - r) * \sigma_t * \bar{\sigma}_t}{RMSE}$$

Equation 2.18 - Theil's U-Statistic: Covariance proportion

being r the correlation between Y_t and f_t.

This means that smaller the bias and variance proportions, better the model forecasting power and error are concentrated in the covariance proportion. Summing all three proportions is equal to 1.

3 Data sets and empirical analysis

The data used in this thesis are time series, representing financial and macroeconomics variables for Ireland, Germany, France, Netherland, Denmark and United Kingdom.

The financial data includes all the SBYs for 10 years' maturity. This data was collected from Bloomberg on 14th of June 2019. The macroeconomic data set includes GDPs, Total Debt Securities, HUR, UK and EU refinancing rates. This data was extracted from Eurostat on 14th of May 2019, except the reference rates which was extracted from Bloomberg on 14th of June 2019 (see SBY codes in the Table 7.1 in the Annex section).

All data have the same time frequency (quarterly) and time frame (March 2000 – March 2019). These countries are related somehow: are geographically closed to each other, are the main ESM of EU, and historically were affected by the last sovereign crisis.

3.1 Sovereign Bond yields

A Yield is a part of an asset return and, normally, is expressed in percentage on an annual basis. Higher risk means higher yields since the investor will require more return to face more risk of losing the money.

This data is quarterly, from 2000's first quarter until 2019's first quarter for the following countries: United Kingdom, Germany, Netherland, Denmark, Ireland and France. The SBY has 10 years maturity (see the Bloomberg codes in the Annex section, Table 7.1). The figures below illustrate the dynamic behaviour of the SBY maturities for each one of the considered countries.

3.1.1 Time series graphs – Analysis

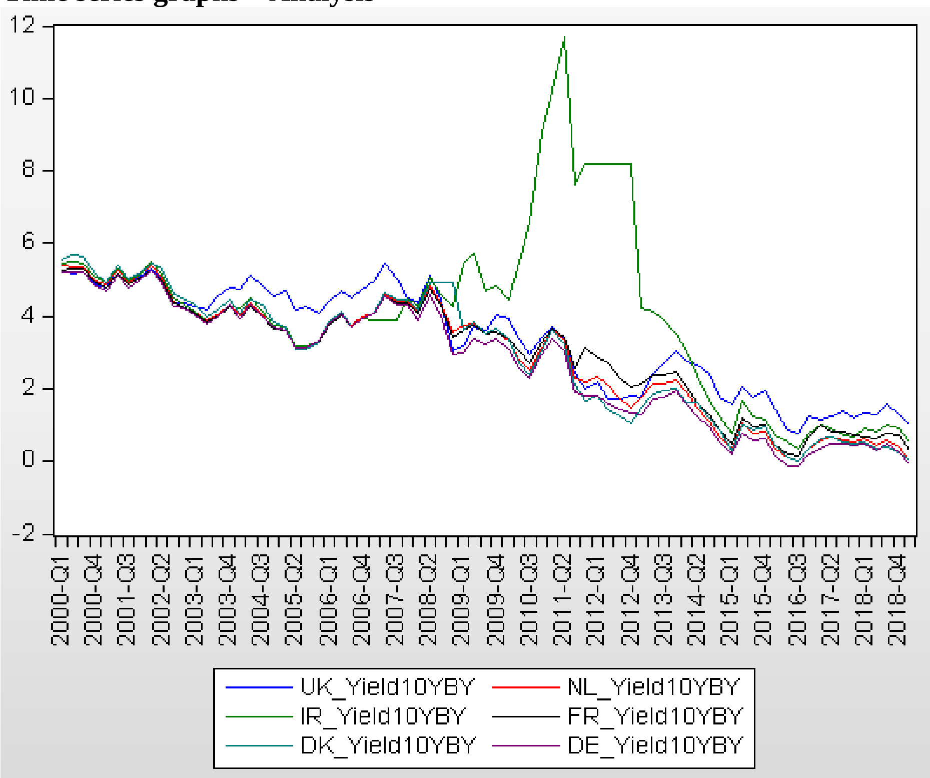


Figure 3.1 - SBY 10 years

As can be seen Irish SBY suffered high increases in the 2010-Q2 and 2015-Q2 period during the sovereign debt crisis, while central ESMs SBY suffered less turbulence and downward trend. The rest of the SBY had the same behaviour since 2000-Q1 until 2009-Q2, although the UK SBY has displayed higher yields between 2002-Q3 and 2007-Q3.

As also can be observed, in the time period between 2014-Q2 and 2015-Q4, we can find some stability and lower yields. After 2015 – Q4 it is interesting also to notice the impact of the Brexit decision process, the referendum on June 2016 and the related subsequent news. Since 2015-Q4 all SBY have been suffering some turbulence, although Brexit was not the only event to trigger more uncertainty (political matters, migration, banking system financial problems, trade war between China and USA) on financial markets.

It is also interesting to witness that the variations along the years, except the sovereign debt crises period have showed the same behaviour and most of the time almost the same pace. This justifies the study of correlations, Granger causalities and Johansen cointegration between all the SBYs. This will allow us to understand if a VAR or VECM models are applicable in order to modelling and perform forecasting. All these questions and observations will be analysed in the following sections.

In the Annex section can be found more detail regarding the general descriptive statistics and the individual plots for the considered dataset (Figure 7.1, Figure 7.2, Figure 7.3, Figure 7.4, Figure 7.5, Figure 7.6, Figure 7.7, Figure 7.8, Figure 7.9, Figure 7.10, Figure 7.11, Figure 7.12, Table 7.2).

3.1.2 Correlations

Regarding the correlation of each SBY time series pairs, the following table contain the parameters:

	DK_YIELD 10YBY	DE_YIELD 10YBY	NL_YIELD 10YBY	FR_YIELD 10YBY	IR_YIELD 10YBY	UK_YIELD 10YBY
DK_YIELD10YBY	1.0000	0.9890	0.9911	0.9786	0.4814	0.9610
DE_YIELD10YBY	0.9890	1.0000	0.9962	0.9885	0.5046	0.9776
NL_YIELD10YBY	0.9911	0.9962	1.0000	0.9951	0.5447	0.9641
FR_YIELD10YBY	0.9786	0.9885	0.9951	1.0000	0.5968	0.9470
IR_YIELD10YBY	0.4814	0.5046	0.5447	0.5968	1.0000	0.4021
UK_YIELD10YBY	0.9610	0.9776	0.9641	0.9470	0.4021	1.0000

Table 3.1 - Correlations between the SBY

It should be considered the following conclusions:

- Very high correlations (above 0.90) between the central ESM (Germany, Netherland, France, Denmark) and between these ones and the UK;
- Irish, French and Dutch SBYs showed high correlation between each other (above 0.70), except the following: French and Irish, Dutch and Irish;
- No negative correlations exist between the SBY;

As mentioned before, correlations symbolize a static approach of studying the SBY impacts, which means that this approach do not offer a dynamic overview of the potential impacts along the time line. This problem can be overcome by performing Granger causalities tests as can be seen in the next section.

3.1.3 Granger Causality

Another aspect analysed in this work were the Granger causalities on this six SBY, considering two different time-lag dimensions' context.

The Granger causality tests were performed with two and four lags, or, in other words, it was studied how a SBY past behaviour can cause another one, considering two quarters and four quarters historical dynamics.

In this test, in order to conclude about causality we analysed the $p-values$. Granger causality is rejected if the $p-values$ are above a particular significance level. In this work it will be analysed up to 10% significance range. Those $p-values$ are displayed below (for two and for four lags in the Figure 3.2 and Figure 3.3, respectively).

Two lags

Pairwise Granger Causality Tests
Date: 08/20/19 Time: 21:10
Sample: 1 77
Lags: 2

Null Hypothesis:	Obs	F-Statistic	Prob.
NL_YIELD10YBY does not Granger Cause UK_YIELD10YBY	75	0.87719	0.4205
UK_YIELD10YBY does not Granger Cause NL_YIELD10YBY		0.22911	0.7958
IR_YIELD10YBY does not Granger Cause UK_YIELD10YBY	75	0.32575	0.7231
UK_YIELD10YBY does not Granger Cause IR_YIELD10YBY		1.62770	0.2037
DK_YIELD10YBY does not Granger Cause UK_YIELD10YBY	75	1.21693	0.3023
UK_YIELD10YBY does not Granger Cause DK_YIELD10YBY		3.54405	0.0342
DE_YIELD10YBY does not Granger Cause UK_YIELD10YBY	75	1.32366	0.2727
UK_YIELD10YBY does not Granger Cause DE_YIELD10YBY		0.24366	0.7844
FR_YIELD10YBY does not Granger Cause UK_YIELD10YBY	75	0.48622	0.6170
UK_YIELD10YBY does not Granger Cause FR_YIELD10YBY		0.06374	0.9383
IR_YIELD10YBY does not Granger Cause NL_YIELD10YBY	75	0.57704	0.5642
NL_YIELD10YBY does not Granger Cause IR_YIELD10YBY		0.75595	0.4734
DK_YIELD10YBY does not Granger Cause NL_YIELD10YBY	75	0.58138	0.5618
NL_YIELD10YBY does not Granger Cause DK_YIELD10YBY		6.61563	0.0023
DE_YIELD10YBY does not Granger Cause NL_YIELD10YBY	75	1.30475	0.2778
NL_YIELD10YBY does not Granger Cause DE_YIELD10YBY		0.70158	0.4993
FR_YIELD10YBY does not Granger Cause NL_YIELD10YBY	75	0.01557	0.9846
NL_YIELD10YBY does not Granger Cause FR_YIELD10YBY		0.72767	0.4867
DK_YIELD10YBY does not Granger Cause IR_YIELD10YBY	75	0.89689	0.4125
IR_YIELD10YBY does not Granger Cause DK_YIELD10YBY		0.29411	0.7461
DE_YIELD10YBY does not Granger Cause IR_YIELD10YBY	75	0.94889	0.3921
IR_YIELD10YBY does not Granger Cause DE_YIELD10YBY		0.81672	0.4460
FR_YIELD10YBY does not Granger Cause IR_YIELD10YBY	75	0.78275	0.4611
IR_YIELD10YBY does not Granger Cause FR_YIELD10YBY		0.99664	0.3743
DE_YIELD10YBY does not Granger Cause DK_YIELD10YBY	75	7.88332	0.0008
DK_YIELD10YBY does not Granger Cause DE_YIELD10YBY		1.31874	0.2740
FR_YIELD10YBY does not Granger Cause DK_YIELD10YBY	75	4.28633	0.0175
DK_YIELD10YBY does not Granger Cause FR_YIELD10YBY		0.57618	0.5647
FR_YIELD10YBY does not Granger Cause DE_YIELD10YBY	75	0.20079	0.8186
DE_YIELD10YBY does not Granger Cause FR_YIELD10YBY		0.84760	0.4328

Figure 3.2 - Pairwise Granger Causality tests for all SBY (2 lags)

Considering two lags it should be noted a high level of Granger Causality on Danish SBY caused by the German, Dutch, French, and English SBYs.

Four lags

Testing Granger causalities with the same significance level but using four lags also showed Granger Causality on Danish SBY caused by the German, Dutch, and English SBYs, except the French one. It is also important to mention that the English SBY cause Granger Causality in the Irish SBY. See Figure 3.3 below:

Pairwise Granger Causality Tests
Date: 08/20/19 Time: 21:32
Sample: 1 77
Lags: 4

Null Hypothesis:	Obs	F-Statistic	Prob.
NL_YIELD10YBY does not Granger Cause UK_YIELD10YBY	73	0.37269	0.8273
UK_YIELD10YBY does not Granger Cause NL_YIELD10YBY		0.87414	0.4845
IR_YIELD10YBY does not Granger Cause UK_YIELD10YBY	73	0.50569	0.7317
UK_YIELD10YBY does not Granger Cause IR_YIELD10YBY		2.38771	0.0601
DK_YIELD10YBY does not Granger Cause UK_YIELD10YBY	73	1.05092	0.3882
UK_YIELD10YBY does not Granger Cause DK_YIELD10YBY		2.18862	0.0802
DE_YIELD10YBY does not Granger Cause UK_YIELD10YBY	73	0.66335	0.6197
UK_YIELD10YBY does not Granger Cause DE_YIELD10YBY		1.16511	0.3346
FR_YIELD10YBY does not Granger Cause UK_YIELD10YBY	73	0.33094	0.8562
UK_YIELD10YBY does not Granger Cause FR_YIELD10YBY		1.69728	0.1616
IR_YIELD10YBY does not Granger Cause NL_YIELD10YBY	73	0.55427	0.6966
NL_YIELD10YBY does not Granger Cause IR_YIELD10YBY		1.32741	0.2694
DK_YIELD10YBY does not Granger Cause NL_YIELD10YBY	73	0.64451	0.6328
NL_YIELD10YBY does not Granger Cause DK_YIELD10YBY		2.49050	0.0518
DE_YIELD10YBY does not Granger Cause NL_YIELD10YBY	73	1.39656	0.2452
NL_YIELD10YBY does not Granger Cause DE_YIELD10YBY		1.03405	0.3966
FR_YIELD10YBY does not Granger Cause NL_YIELD10YBY	73	0.24046	0.9144
NL_YIELD10YBY does not Granger Cause FR_YIELD10YBY		0.79594	0.5322
DK_YIELD10YBY does not Granger Cause IR_YIELD10YBY	73	1.23625	0.3045
IR_YIELD10YBY does not Granger Cause DK_YIELD10YBY		0.38532	0.8184
DE_YIELD10YBY does not Granger Cause IR_YIELD10YBY	73	1.68756	0.1638
IR_YIELD10YBY does not Granger Cause DE_YIELD10YBY		0.97182	0.4292
FR_YIELD10YBY does not Granger Cause IR_YIELD10YBY	73	0.96797	0.4313
IR_YIELD10YBY does not Granger Cause FR_YIELD10YBY		1.32701	0.2695
DE_YIELD10YBY does not Granger Cause DK_YIELD10YBY	73	3.24334	0.0174
DK_YIELD10YBY does not Granger Cause DE_YIELD10YBY		0.77906	0.5429
FR_YIELD10YBY does not Granger Cause DK_YIELD10YBY	73	1.51123	0.2095
DK_YIELD10YBY does not Granger Cause FR_YIELD10YBY		0.40146	0.8069
FR_YIELD10YBY does not Granger Cause DE_YIELD10YBY	73	0.30758	0.8719
DE_YIELD10YBY does not Granger Cause FR_YIELD10YBY		0.78150	0.5414

Figure 3.3 - Pairwise Granger Causality tests for all SBY (4 lags)

In conclusion:

- The SBY (considering the geographical criteria) displayed that Danish one can be Granger caused by almost all other SBYs studied, regardless of the lags used;
- The UK showed also some influence (Granger causality) on the other SBY.

3.1.4 Unit root tests – stationarity

Regarding the unit root tests, or in other words, analysing the stationarity of the time series, it was performed three tests, namely, ADF, KPSS and PP, for the series in level (using all data as is) and in first differences. All these tests were performed by including trend and intercept and using 14 lags for the ADF approach. The main purpose by performing these tests is to check if the time series is stationary. Otherwise all the statistical analyses (for example, the $p-values$, or f-statistics) can be biased.

Even though a time series is not stationary it can be adjusted by doing the simple first differences or the logarithmic first differences. In this case, since a yield can be negative it cannot be subject to logarithm, so the first difference will be performed.

The conclusions were based on tests of a significance at 5% level and considering the best two results of the three methods used.

	Level				1st diferencce			
	ADF Test H0: unit root Rejected if: p-value < 5%	KPSS test H0: is stationary Rejected if: LM test > 0,146 (5%) LM test > 0,216 (1%)	Phillips-Perron H0: unit root Rejected if: p-value < 5%	Non-stationary (unit root)	ADF Test H0: unit root Rejected if: p-value < 5%	KPSS test H0: is stationary Rejected if: LM test > 0,146 (5%) LM test > 0,216 (1%)	Phillips-Perron H0: unit root Rejected if: p-value < 5%	Non-stationary (unit root)
dk_yield10yby	0.202	0.134	0.218	S	0.000	0.191	0.000	N
de_yield10yby	0.137	0.151	0.152	S	0.000	0.095	0.000	N
nl_yield10yby	0.258	0.165	0.272	S	0.000	0.077	0.000	N
fr_yield10yby	0.237	0.176	0.268	S	0.000	0.083	0.000	N
uk_yield10yby	0.030	0.160	0.080	S	0.000	0.092	0.000	N
ir_yield10yby	0.785	0.176	0.785	S	0.000	0.072	0.000	N

Table 3.2 - Stationarity tests

The conclusions are that all SBY time series are non-stationary in level. After stabilizing the series, by performing first differences, all the time series become stationary, which means that the time series are integrated of order one.

3.1.5 Cointegration analysis

The cointegration analysis will be employed by taking into consideration the previous results, namely the same integration order for all variables ($I(1)$) and the Granger Causality between some of the SBY's.

In order to proceed with this analysis, we use the Johansen methodology. The specification of the Johansen Cointegration Trace and Maximum Eigenvalue tests foresees intercept and trend (but no intercept in VAR), with two and four lags for 5% and 10% significant level.

Two lags

In what follows we analyse the existence of cointegrating vectors between the SBY, reminding, United Kingdom, Germany, Denmark, Netherland and France, using two and four lags.

Johansen Cointegration Test

Date: 06/11/20 Time: 01:45
Sample (adjusted): 4 77
Included observations: 74 after adjustments
Trend assumption: Linear deterministic trend (restricted)
Series: UK_YIELD10YBY NL_YIELD10YBY IR_YIELD10YBY FR_YIELD10YBY DK_YIELD10YBY DE_YIELD10YBY
Lags interval (in first differences): 1 to 2

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.562000	141.9105	117.7082	0.0006
At most 1	0.465443	80.82078	88.80380	0.1640
At most 2	0.166251	34.47331	63.87610	0.9673
At most 3	0.121110	21.01843	42.91525	0.9391
At most 4	0.095669	11.46534	25.87211	0.8473
At most 5	0.052926	4.023953	12.51798	0.7386

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.562000	61.08971	44.49720	0.0004
At most 1 *	0.465443	46.34747	38.33101	0.0049
At most 2	0.166251	13.45488	32.11832	0.9819
At most 3	0.121110	9.553086	25.82321	0.9743
At most 4	0.095669	7.441389	19.38704	0.8684
At most 5	0.052926	4.023953	12.51798	0.7386

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Figure 3.4 - Johansen Cointegration test for the SBY with two lags

The outputs exhibited in Figure 3.4, shows the existence of at least 1 cointegrating vectors for a 5% and 10% significance level based on the Trace statistics, for 10 years SBY maturity. As for the Maximum Eigenvalue, it indicates up to 2 cointegrating equations for a 5% significance level.

Four lags

Repeating the same exercise, but using 4 lags instead of 2, Johansen Cointegration tests will suggest more cointegrating vectors compared to the 2 lags tests, as the figure below shows:

Johansen Cointegration Test

Date: 06/11/20 Time: 01:40
Sample (adjusted): 6 77
Included observations: 72 after adjustments
Trend assumption: Linear deterministic trend (restricted)
Series: UK_YIELD10YBY NL_YIELD10YBY IR_YIELD10YBY FR_YIELD10YBY DK_YIELD10YBY DE_YIELD10YBY
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.618548	178.9012	117.7082	0.0000
At most 1 *	0.567454	109.5098	88.80380	0.0007
At most 2	0.220800	49.16909	63.87610	0.4513
At most 3	0.190552	31.20602	42.91525	0.4323
At most 4	0.130064	15.98504	25.87211	0.4938
At most 5	0.079353	5.952856	12.51798	0.4663

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.618548	69.39140	44.49720	0.0000
At most 1 *	0.567454	60.34075	38.33101	0.0000
At most 2	0.220800	17.96308	32.11832	0.8027
At most 3	0.190552	15.22098	25.82321	0.6146
At most 4	0.130064	10.03218	19.38704	0.6154
At most 5	0.079353	5.952856	12.51798	0.4663

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

Figure 3.5 - Johansen Cointegration test for the SBY with four lags

The Figure 3.5 showed up to 2 cointegrating vectors for a 5% and 10% significance threshold based on Trace and on the Maximum Eigenvalue tests.

The previous tests exhibited just up to 1 cointegrating vector.

In a nutshell, the Johansen cointegration methodology based on the Trace and Maximum Eigenvalue statistics showed that:

- All outputs suggested, at least 1 cointegrating vector for a 5% significance level;
- More lags led to more cointegrating vectors, which can suggest more linear long-term relationship.

3.1.6 Vector Error Correction Model

After checking traces of existence of Cointegration equations (CEQ for simple presentation), lets see how they look by using the VEC modeling. It was used the same lag criteria, which was considered in the Granger causality and Johansen tests.

One of the drivers for this test is the optimal lags number for the endogenous variables. The optimal lags can be defined by performing a standard VAR test and observe the Lag Structure. The output is available in the Figure 7.13, in Annex section. Since VECM implicitly omits one lag we will use 2 lags in the VECM analysis, which goes towards the Lag criteria results.

The first CEQ and the corresponding error correction equation (VECM) for the English SBY as the target variable (using all the other SBY) is (detailed in Annex, Figure 7.14) given by:

$$CEQ_{t-1} = 1.000UK_{t-1} - 29.4685NL_{t-1} + 0.2203IR_{t-1} + 13.9836FR_{t-1} + 10.3358DK_{t-1} + 5.7531DE_{t-1} + 0.0902trend - 9.9363$$

Equation 3.1 - Cointegrating equation for the English SBY

$$\Delta UK_t = -0.1230CEQ_t - 0.3251UK_{t-1} - 0.6718UK_{t-2} - 1.21310NL_{t-1} - 0.8906NL_{t-2} - 0.0504IR_{t-1} - 0.0698IR_{t-2} + 0.7463FR_{t-1} + 0.2181FR_{t-2} + 0.5234DK_{t-1} + 0.2426DK_{t-2} + 0.4037DE_{t-1} + 08892DE_{t-2} - 0.0544$$

Equation 3.2 – Estimated VECM with the English SBY as the target variable

The Figure 7.18 and Figure 7.19 (in the Annex) represents the OLS output and the Residual serial correlation (respectively) test for this VECM equation. Some conclusions can be taken from this output:

- The $C(1)$ coefficient is -0.1230, which is the velocity adjustment parameter. This means that, at each time step, around 12% should be corrected in order to the model dynamics maintain close to the long-range equilibrium relation given by the cointegration vector CEQ_{t-1};
- It is very important that the C(1) value is negative (the adjustment points to the correct convergence direction) and it is statistically significant at 5% level ($p - value$ of 0.0157 below 0.05);
- The coefficient $C(3)$ is also important meaning that the past values of the English SBY (with 2 lags) seems to be statistically relevant since the $p - value$ is lower than 5%.
- The Residual serial correlation test shows a very high Chi-Square (0.9402) $p - value$ result (above 10% of significance level) which means that there is no evidence of residual correlation.

The second CEQ and the corresponding error correction equation will have the German SBY as the target variable (using all the rest of the SBY) and are represented by the expressions below (detailed in the Annex section in Figure 7.20).

$$CEQ_{t-1} = 1.000DE_{t-1} - 5.1221NL_{t-1} + 0.0383IR_{t-1} + 2.4306FR_{t-1} + 1.7966DK_{t-1} + 0.1738UK_{t-1} + 0.0157trend - 1.7271$$

Equation 3.3 - Cointegrating equation for the German SBY

$$\Delta DE_t = -0.6701CEQ_t - 0.1859UK_{t-1} - 0.6201UK_{t-2} - 0.9465NL_{t-1} - 0.9356NL_{t-2} - 0.0387IR_{t-1} - 0.0119IR_{t-2} + 0.8700FR_{t-1} + 0.3168FR_{t-2} + 0.5442DK_{t-1} + 0.1856DK_{t-2} - 0.1640DE_{t-1} + 0.7943DE_{t-2} - 0.0734$$

Equation 3.4 - Estimated VECM with the German SBY as the target variable

Regarding the OLS equation and Residual serial correlation test for the German VECM model (see the Annex section, Figure 7.24 and Figure 7.25, respectively) there are also some notes to take into account:

- The coefficient $C(1)$ is -0.6701 and it is the velocity adjustment parameter, which means that at each time step, around 67% should be corrected in order to the model dynamics maintain close to the long-range equilibrium relation given by the cointegration vector CEQ_{t-1};
- The alpha value is negative and it is also statistically significant at 5% level ($p - values$ of 0.0112);
- The coefficients $C(13)$ which refers to the impact of the English SBY (with 2 lags) should be considered statistically significant since the $p - value$ is lower than 5%;
- The $C(5)$ and $C(8)$ coefficients $p - value$s, which corresponds to the Dutch SBY (with 1 lag) and French SBY (with two lags), are close to an acceptable significance level of 10%, although a bit higher than it;
- The Residual serial correlation test displayed a very high Chi-Square (0.8431) $p - value$ result (above 10% of significance level) which means that there is no evidence of residual correlation.

In conclusion, both VEC models showed some evidences of correlation in a long-run period, namely the relation between the lagged English SBY and the German SBY. Moreover, there was no residual correlation meaning that the model will not be biased due to the residuals. In these outputs was difficult to find evidences of all the Granger causality cases that were found along the Granger Causality tests.

3.2 Macroeconomic Data

In the other sections it was observed that, apart from high correlations, there were indications of causality and cointegration between SBY, which suggests the presence of correlation in long-term.

The major Brexit's studies attested that investment, GDP and other macroeconomic variables would be affected negatively. The purpose of the application of macroeconomic variables is to link the Brexit studies and SBYs.

The first macroeconomic variable chosen was GDP, one of the main aggregates on national accounts, which captures the economic activity within a territory and in a specific time period. The components of this indicator are Private and Public consumption, Capital and Investment and Net exportations (difference between Exports and Imports). It also represents the value of the final goods and services produced by a country, net by the imports. This metric is seasonally adjusted. The Debt securities as percentage of GDP is a ratio ("country"_ DEBTSEC) between the total debt securities for the general government sector (on the numerator) and the total GDP at current prices (on denominator). These data were collected from Eurostat.

HUR ("country"_ HUR) represents the unemployed person who is available and has conditions to work (is in a working age) and is without work despite the specific measures taken to find work. This rate is one of the most uniform way to compare unemployment rate internationally. The HUR was collected from OECD.

All these Macroeconomic variables were collected taking into account the time line 2000's first quarter until 2019's first quarter (except the DEBTSEC, since it there was no available data).

Reference interest rate on a country is the rate of interbank fund rate. The UK's rate as well as the EU's rate were pulled from Bloomberg:

- EURR002W Index – European Central Bank reference rate;
- UKBRBASE Index - Bank of England Official Bank Rate.

The purpose of the next sections is to perform the same econometric analysis performed before, but, this time, between the SBY and their respective macroeconomic variables (debt securities as % of GDP, the HUR) and reference rates described above.

3.2.1 Correlations

As seen before in the Correlation section, some SBY displayed high correlations.

In the Table 3.3 below are presented the correlations between the SBY and the macroeconomic variables. In "green" and "yellow" are marked the correlations between 0.80 and 1 and between -1 and -0.80, respectively, showing the strongest relations between the considered variables.

	UK_HUR	UK_DEBTSE	UKBRBASE_	NL_DEBTSE	NL_HUR	IR_HUR	IR_DEBTSE	FR_HUR	FR_DEBTSE	DK_HUR	DK_DEBTSE	DE_HUR	DE_DEBTSE	EURR002W_
UK_YIELD10	-0.1213	-0.9239	0.8678	-0.7306	-0.4768	-0.5112	-0.7394	-0.6214	-0.9391	-0.5784	0.2889	0.8672	-0.6445	0.8719
UK_HUR	1.0000	0.3288	-0.4499	0.4350	0.2652	0.8932	0.6065	0.2809	0.2313	0.6739	0.1031	-0.1448	0.6603	-0.1966
UK_DEBTSE	0.3288	1.0000	-0.9146	0.8446	0.5796	0.6950	0.8930	0.7194	0.9761	0.7142	-0.2544	-0.9019	0.7875	-0.8763
UKBRBASE_	-0.4499	-0.9146	1.0000	-0.7847	-0.5072	-0.7701	-0.8971	-0.6423	-0.9188	-0.7975	0.1772	0.7848	-0.7906	0.9187
NL_YIELD10	0.0085	-0.9041	0.8133	-0.7189	-0.5844	-0.4121	-0.6825	-0.6375	-0.9429	-0.5238	0.3402	0.8007	-0.6425	0.8978
NL_DEBTSE	0.4350	0.8446	-0.7847	1.0000	0.7884	0.7144	0.8795	0.9037	0.7725	0.7867	0.1311	-0.6735	0.7198	-0.7828
NL_HUR	0.2652	0.5796	-0.5072	0.7884	1.0000	0.4909	0.5809	0.7240	0.5678	0.6023	0.0018	-0.2624	0.6984	-0.6604
IR_YIELD10	0.6867	-0.2311	0.1193	-0.1216	-0.2430	0.4173	-0.0093	-0.2176	-0.3299	0.2616	0.3384	0.2898	0.0837	0.3468
IR_HUR	0.8932	0.6950	-0.7701	0.7144	0.4909	1.0000	0.8602	0.5228	0.6187	0.8656	-0.0197	-0.4925	0.8673	-0.5752
IR_DEBTSE	0.6065	0.8930	-0.8971	0.8795	0.5809	0.8602	1.0000	0.7412	0.8322	0.8273	-0.0393	-0.7636	0.8045	-0.7836
FR_YIELD10	0.0655	-0.8799	0.7920	-0.6949	-0.5742	-0.3607	-0.6544	-0.6203	-0.9281	-0.4764	0.3591	0.7826	-0.6103	0.8909
FR_HUR	0.2809	0.7194	-0.6423	0.9037	0.7240	0.5228	0.7412	1.0000	0.6520	0.6844	0.2702	-0.5657	0.5185	-0.6999
FR_DEBTSE	0.2313	0.9761	-0.9188	0.7725	0.5678	0.6187	0.8322	0.6520	1.0000	0.6650	-0.3510	-0.8571	0.7793	-0.9245
DK_YIELD10	-0.0498	-0.9171	0.8234	-0.7269	-0.5963	-0.4586	-0.6994	-0.6540	-0.9507	-0.5694	0.3204	0.7977	-0.6750	0.8997
DK_HUR	0.6739	0.7142	-0.7975	0.7867	0.6023	0.8656	0.8273	0.6844	0.6650	1.0000	0.2498	-0.4782	0.7422	-0.7287
DK_DEBTSE	0.1031	-0.2544	0.1772	0.1311	0.0018	-0.0197	-0.0393	0.2702	-0.3510	0.2498	1.0000	0.2496	-0.3721	0.1639
DE_YIELD10	-0.0519	-0.9242	0.8444	-0.7445	-0.5820	-0.4654	-0.7224	-0.6433	-0.9559	-0.5528	0.3405	0.8244	-0.6738	0.9037
DE_HUR	-0.1448	-0.9019	0.7848	-0.6735	-0.2624	-0.4925	-0.7636	-0.5657	-0.8571	-0.4782	0.2496	1.0000	-0.5091	0.6843
DE_DEBTSE	0.6603	0.7875	-0.7906	0.7198	0.6984	0.8673	0.8045	0.5185	0.7793	0.7422	-0.3721	-0.5091	1.0000	-0.7239
EURR002W_	-0.1966	-0.8763	0.9187	-0.7828	-0.6604	-0.5752	-0.7836	-0.6999	-0.9245	-0.7287	0.1639	0.6843	-0.7239	1.0000

Table 3.3 - Correlations between SBY and the macroeconomic variables

Mixing macroeconomic variables and SBY high correlations stands out:

- The negative correlation between the UK_DEBTSEC and the English, German, Dutch, French 10 years SBY, German HUR and English reference rate;
- The negative correlation between UKBRBASE and UK, NL, IR and FR DEBTSEC; a positive one with the European reference rate, the English, Danish and German SBYs;
- The positive correlation between EURR002W and UK, NL, FR, DK, DE SBY;
- Positive correlations between the IR_HUR and DE_HUR with UK HUR; The same is observed comparing the NL and FR HUR;
- A positive correlation is also observed between the almost all the other HUR and the DEBTSEC.

3.2.2 Granger Causality

The same tests were performed to find if it is rejected or not the existence of Granger Causalities between these variables, excluding the Granger Causality between the SBY. As done before, this test will be performed using 2 and 4 lags, and for 10% of significance level. All the outputs can be seen in the Annex section.

Two lags

Analysing these results we found 163 Granges causality relations (see Figure 7.26, Figure 7.27, Figure 7.28 and Figure 7.29 in Annex chapter) at a significance level of 10%, of which:

- The UK reference rate “UKBRBASE_INDEX” causes Granger on 13 other variables, highlighting the SBY (Dutch and Danish), Debt securities in % of GDP (English, Irish, Dutch, German and French), Unemployment rate (English, Irish, German and French), and EU reference rate;
- There were Granger causality of UK Debt securities as % of GDP on 16 other variables, pointing out SBYs (English, French, Dutch, Danish and German), the Debt Securities in % of GDP (German, Danish, Dutch, French and Irish) the Unemployment rate (Irish, French, Danish and German) and European reference rate;
- The English SBY Granger cause 5 other variables, namely: the English bank and European reference rate, Irish, French and German Unemployment rate;
- Excluding the English reference rate, the English variables Granger cause other 31 variables of a total 163 rejections;
- 21 Granger causality relations exist on English variables caused by others (except English ones), highlighting 5 on English reference rate, 7 on English Debt Securities in % of GDP (mainly caused by German, Danish, French and Irish Debt Securities % GDP) and 7 HUR, 2 cases on English SBY (German HUR and French debt in % of GDP);
- There were 21 Granger causality of German variables on other ones (German included), mainly over the HURs (7 cases, all sovereign, except the Irish), 5 over SBY (only German, France, Dutch, Danish and English) and other 5 over Debt Securities as % of GDP;
- Among the 21 cases of Granger Causalities by the German variables, the German HUR was the one with more Granger Causality rejection cases (8 of 21 total);
- It also should be highlighted that there were 13 cases of Granger causality over the German Debt securities in % of GDP (excluding the German causes) of 26 total.

Four lags

Using four lags (see Figure 7.30, Figure 7.31 and Figure 7.32 in the Annex chapter) and comparing these results with the 2 lags case, there were 135 Granger Causality relations instead of 161. However, it should be referred:

- Granger causality of the English reference rate on 8 other variables (instead of 13);
- The English SBY showed more cases of Granger causality rejections (7 instead of 5).
- Granger causality can be found in all 5 SBY and in the Debt Securities in % of GDP (French, Danish, Irish, German), all caused by the English Debt Securities in % of GDP (14 cases in total).
- Excluding the English reference rate, English variables Granger cause other 27 variables;
- There were 20 cases of Granger Causality in the English variables caused by other ones (except other English variables), of which 5 caused English reference rate, 6 English Debt

Securities in % of GDP (caused by the Danish, French and Irish HUR and Debt Securities in % GDP), 6 caused HUR, and 3 English SBY (caused by the German, French and Irish Debt Securities in % of GDP);

- On the other hand, German variables showed 16 Granger causality cases highlighting 6 over HURs (German, Irish, French and English) and 4 over SBY (German, Danish, Dutch and English);
- 25 cases of Granger Cause German variables (expect the caused by German variables), of which 12 cause German Debt Securities as % of GDP (caused by Danish, French, Irish, Dutch and English variables and by both reference rates).

In conclusion, the results suggest high level of Granger causality between the macroeconomic variables and the SBYs. Comparing the tests with 4 lags and with 2 lags, it was observed more Granger causalities using 2 lags, highlighting on the English reference rate and German Granger Causalities. This could mean that impacts can occur more often in shorter terms comparing with the longer ones.

3.3 Regressions

All the previous tests allowed to confirm the static and dynamic relation between SBY and macroeconomic variables. It is now important to test if a statistically significant regression can be generated mixing the variables analysed in the last sections.

Despite the wide range of SBYs studied, there are two that should be highlighted – the German and English SBYs. The first one is commonly used as a benchmark or as a proxy of risk free rate. The risk free rate is also frequently used for investments or portfolio assessment and theory/academic purposes. The second one is one of the main drivers of this thesis and for this reason it also should be considered.

Several equations were tested and after analysing the main output indicators as well as its residuals two models were chosen, one for each SBY. Below, it is presented a resume of all outputs being the detailed ones in the Annex chapter (for Equation A: Figure 7.33, Figure 7.34, Figure 7.35, Figure 7.36; for Equation B: Figure 7.37, Figure 7.38, Figure 7.39, Figure 7.40):

	Indicators / Tests	de_yield10yby c de_debtsec_gdp uk_yield10yby ir_yield10yby nl_yield10yby nl_debtsec_gdp dk_debtsec_gdp **A**		uk_yield10yby c de_yield10yby fr_yield10yby de_debtsec_gdp dk_debtsec_gdp ir_debtsec_gdp ir_hur uk_hur **B**		Parameters
Main output	R^2	0.9986	Comply	0.9883	Comply	1
	Prob stat.	0.0000	Comply	0.0000	Comply	0
	Durbin Watson stat.	1.2551	Partialy comply	1.2032	Partialy comply	1 - 3 (2)
	Akaike / Schwarz	-2.5080 / -2.2933	Comply	-0.6694 / -0.4241	Partialy comply	< -∞
Residuals	Norm. (prob.)	0.1455	Comply	0.4812	Comply	> 0.10
	Mean	7.41E-17	Comply	-7.07E-16	Comply	0
	Skewness	-0.3589	Partialy comply	-0.1879	Partialy comply	0
	Kurtosis	3.838	Partialy comply	2.4337	Partialy comply	3
	BG serial Corr LM test	0.0081	Not comply	0.0023	Not comply	> 0.10
	BPG (heteros.)	0.1348	Comply	0.3763	Comply	> 0.10
	Correlogram resid.	trend to 0	Comply	trend to 0	Comply	0
	Correlogram resid. sqrd.	trend to 0	Comply	trend to 0	Comply	0

✓ Comply ✓ Partialy comply ✗ Not comply

Figure 3.6 - Regression´s outputs resume for the two models

Comments:

- All the main outputs showed a strong and well behaved models, with high R^2, low Akaike and Schwarz information criteria values and no model was globally rejected, based on the F-statistics;
- None of the variables chosen for each model were rejected at a 5% significant level;
- The Durbin-Watson showed values between 1 and 3, although very close to the left threshold, which means that some first order correlation between residuals may exist;
- The Skewness and the Kurtosis of the Residuals are quite close to the normality distribution standards;
- Notwithstanding some of the tests failed, namely the residuals correlations. All BG LM tests failed and all DW tests, despite within the boundaries, showed figures well close to the lower limit which could indicate correlated errors.

In order to solve the dependence problem an autoregressive term can be added, performing a new dynamic model, but this will be not done here, being part of future projects.

In terms of equations for model A and B:

$$EQ_A: DE_{\mathrm{SBY}} = 0.6026 - 0.9280DE_{DebtSecGDP} + 0.3027UK_{SBY} + 0.0184IR_{SBY} + 0.6767NL_{SBY} - 1.5583NL_{DebtSecGDP} + 0.5420DK_{DebtSecGDP}$$

Equation 3.5 - Model A equation

$$EQ_B: UK_{\mathrm{SBY}} = -3.4737 + 1.3692DE_{SBY} - 0.3280FR_{SBY} + 10.9900DE_{DebtSecGDP} + 1.4101DK_{DebtSecGDP} + 1.1576IR_{DebtSecGDP} - 8.4567IR_{HUR} - 22.8039UK_{HUR}$$

Equation 3.6 - Model B equation

Comments:

- The German SBY seems to react positively if the English, Irish and Dutch SBY increases, which makes sense economically and financially;
- On the other hand, it decreases if German and Dutch Debt in % of GDP increases. It is interesting also to notice the positive impact by the increase of the Danish debt.
- Regarding the English SBY, reacts positively to the German SBY and also to the German, Danish and Irish Debt in % of GDP.
- The English SBY decreases if the Irish and English HUR increases. The increase of the French SBY causes negative impacts to the English SBY.

Taking into account the potential negative impacts in terms of GDP and unemployment referred in the Introduction chapter, the regressions show that the English and German SBY reacts to these variations.

4 Forecasting

Forecasts are frequently used by finance analysts with the purpose of anticipating what may happen in the future. Using this information investors and analysts reduce risk and uncertainty to take decisions.

Using in-sample approach the forecasts was performed for the last 8 observations based on all historic data, in others words, the regression model will be used to predict the German and the English SBY figures for the 1Q17 - 4Q18 period (E-views adjusts the last observation). In the Annex section, Figure 7.41 and Figure 7.42, are presenting the detailed outputs related to the "in-sample" forecasts.

	Indicators / Tests	de_yield10yby c de_debtsec_gdp uk_yield10yby ir_yield10yby nl_yield10yby nl_debtsec_gdp dk_debtsec_gdp (A)		uk_yield10yby c de_yield10yby fr_yield10yby de_debtsec_gdp dk_debtsec_gdp ir_debtsec_gdp ir_hur uk_hur (B)		Parameters
Forecasting for the last 8 observations	Root means squared error	0.0633	✓ Comply	0.1900	✓ Partialy comply	0
	Theil Inequality Coef.	0.0770	✓ Comply	0.0708	✓ Comply	0
	Bias Proportion	0.0105	✓ Comply	0.1033	✓ Partialy comply	0
	Variance Proportion	0.0235	✓ Comply	0.0679	✓ Comply	0
	Covariance Proportion	0.9660	✓ Comply	0.8288	✓ Comply	1

✓ Comply ✓ Partialy comply ✗ Not comply

Figure 4.1 - Forecast outputs "in-sample" for the models

As for the out-of-sample predictions (more detail in the Annex chapter, Figure 7.43 and Figure 7.44) and considering the same forecasted quarters, the models will use the 1Q00 to 4Q16 data and will forecast the figures for 1Q17 - 4Q18 period). The output is the following:

	Indicators / Tests	de_yield10yby c de_debtsec_gdp uk_yield10yby ir_yield10yby nl_yield10yby nl_debtsec_gdp dk_debtsec_gdp (A)		uk_yield10yby c de_yield10yby fr_yield10yby de_debtsec_gdp dk_debtsec_gdp ir_debtsec_gdp ir_hur uk_hur (B)		Parameters
Forecasting for the last 8 observations	Root means squared error	0.0754	✓ Comply	0.2205	✓ Partialy comply	0
	Theil Inequality Coef.	0.0890	✓ Comply	0.0800	✓ Comply	0
	Bias Proportion	0.1686	✓ Partialy comply	0.3640	✓ Partialy comply	0
	Variance Proportion	0.0112	✓ Comply	0.0451	✓ Comply	0
	Covariance Proportion	0.8201	✓ Comply	0.5909	✓ Partialy comply	1

✓ Comply ✓ Partialy comply ✗ Not comply

Figure 4.2 - Forecast outputs "out of sample" for the models

Comments:

- Comparing the Figure 4.1, Figure 4.2, in model A no significant changes occur, although the Bias proportion suffered an increase. As for model B, it was detected more differences. However, in both cases, the differences do not compromise the use of those models;

- Both models displayed a satisfactory outputs considering the in-sample and out-of-sample forecasting approaches, notwithstanding, the equation B displayed higher RMSE and Bias proportion and a lower covariance coefficient in the out-of-sample model;
- Analysing the outputs (Figure 7.41, Figure 7.42, Figure 7.43 and Figure 7.44) the forecasted line (blue one) did not cross the 2 standards deviations (red lines), passing through them at a confidence level of 95%. This means that the regressions reproduces forecasted SBY figures very close to the observed ones.

In Table 4.1 and Table 4.2 we can take a closer look to the outputs in order to compare 8 forecasted figures using the "in-sample" and "out-of-sample" with the real figures:

Period	DE_YIELD10F (1)	DE_YIELD10YBY (2)	Difference (1-2)
2016-Q3	-0.1190	-0.1190	
2016-Q4	0.2080	0.2080	
2017-Q1	0.3200	0.3280	-0.0080
2017-Q2	0.4181	0.4660	-0.0479
2017-Q3	0.4260	0.4640	-0.0380
2017-Q4	0.3345	0.4270	-0.0925
2018-Q1	0.4959	0.4970	-0.0011
2018-Q2	0.3619	0.3020	0.0599
2018-Q3	0.5551	0.4700	0.0851
2018-Q4	0.3364	0.2420	0.0944

Table 4.1 - Model A: Forecast "in-sample" vs Real figures

Period	UK_YIELD 10F (1)	UK_YIELD 10YBY (2)	Difference (1-2)
2016-Q3	0.7460	0.7460	
2016-Q4	1.2390	1.2390	
2017-Q1	1.2309	1.1390	0.0919
2017-Q2	1.5456	1.2570	0.2886
2017-Q3	1.5111	1.3650	0.1461
2017-Q4	1.4106	1.1900	0.2206
2018-Q1	1.5326	1.3500	0.1826
2018-Q2	1.2554	1.2780	-0.0226
2018-Q3	1.4167	1.5730	-0.1563
2018-Q4	1.0147	1.2770	-0.2623

Table 4.2 - Model B: Forecast "in-sample" vs Real figures

As for the "in-sample" forecasts and taking a look to the model A, it showed some divergences, sometimes, very slight ones (in the 2017Q1 and 2018 Q1). In the 2017's and 2018's last quarter it showed higher differences, which means less accurate forecast.

Concerning the model B, it displayed higher divergences (especially in the 2017´s and 2018´s last quarters) comparing to the model A. It is interesting to notice that both models followed the same dynamics comparing the real figures (increases and decreases) along the 8 quarters which can lead to the conclusion about the good quality of these two models. Positive and negative difference values mean that the forecast is undervalued or overvalued.

Regarding the "out-of-sample" forecast, the results are the presented in the following tables Table 4.3 and Table 4.4:

Period	DE_YIELD10F (1)	DE_YIELD10YBY (2)	Difference (1-2)
2016-Q3	-0.1190	-0.1190	
2016-Q4	0.2080	0.2080	
2017-Q1	0.3359	0.3280	0.0079
2017-Q2	0.4343	0.4660	-0.0317
2017-Q3	0.4471	0.4640	-0.0169
2017-Q4	0.3559	0.4270	-0.0711
2018-Q1	0.5202	0.4970	0.0232
2018-Q2	0.3914	0.3020	0.0894
2018-Q3	0.5872	0.4700	0.1172
2018-Q4	0.3717	0.2420	0.1297

Table 4.3 - Model A: Forecast "out-of-sample" vs Real figures

Period	UK_YIELD10F (1)	UK_YIELD10YBY (2)	Difference (1-2)
2016-Q3	0.7460	0.7460	
2016-Q4	1.2390	1.2390	
2017-Q1	1.2981	1.1390	0.1591
2017-Q2	1.6136	1.2570	0.3566
2017-Q3	1.5781	1.3650	0.2131
2017-Q4	1.4782	1.1900	0.2882
2018-Q1	1.6047	1.3500	0.2547
2018-Q2	1.3333	1.2780	0.0553
2018-Q3	1.4935	1.5730	-0.0795
2018-Q4	1.0936	1.2770	-0.1834

Table 4.4 - Model B: Forecast "out-of-sample" vs Real figures

Concerning the "out-of-sample" forecasts, both displayed higher differences comparing with the "in-sample" approach. Nevertheless, both models also showed the same dynamics comparing with the real figures (increases and decreases) along the 8 observations. Again, we still can conclude that both models seem to have good predictive abilities.

Another interesting exercise was to use the two models, A and B, and simulate the German and English SBY based on more recent data for the equation's dependent variables. The new data 2019Q2, 2019Q3 and 2019Q4 were collected from the same source (Bloomberg, Eurostat and OECD).

The purpose was to compare both SBY, the real figures for these three 2019's quarters and check the divergences with the calculated values with base on models A and B. The following Table 4.5 and Table 4.6 contains the estimated equations A and B, respectively. In these tables can be found the coefficients and the real variables. In the end of each, it sums up after calculating the product between the coefficients and the variables ("SBY_YIELD10YBY Forecast") and compare with the real ones.

DE_YIELD10YBY	Coefficients	2019Q2	2019Q3	2019Q4
C	0.6026	0.6026	0.6026	0.6026
DE_DEBTSEC_GDP	-0.9280	0.4490	0.4480	0.4390
UK_YIELD10YBY	0.3027	0.8330	0.4880	0.8220
IR_YIELD10YBY	0.0184	0.1740	-0.0350	0.1190
NL_YIELD10YBY	0.6767	-0.1600	-0.4290	-0.0600
NL_DEBTSEC_GDP	-1.5583	0.4060	0.3880	0.3830
DK_DEBTSEC_GDP	0.5420	0.2610	0.2580	0.2480
DE_YIELD10YBY Forecast (1)		**-0.1582**	**-0.4212**	**-0.0568**
DE_YIELD10YBY Real (2)		**-0.3270**	**-0.5710**	**-0.1850**
Difference (2-1)		***-0.1688***	***-0.1498***	***-0.1282***

Table 4.5 - Model A with more recent data vs real figures

As can be seen, there were some divergences, nevertheless the forecasted figures also showed the same dynamic (decrease and increase) and the differences also displayed a downward trend along the three 2019's quarters, which is also positive indicator concerning the forecast quality of the equation A.

The following table is regarding the model B for English SBY:

UK_YIELD10YBY	**Coefficients**	**2019Q2**	**2019Q3**	**2019Q4**
C	-3.4737	-3.4737	-3.4737	-3.4737
DE_YIELD10YBY	1.3692	-0.3270	-0.5710	-0.1850
FR_YIELD10YBY	-0.3280	0.0099	-0.2443	0.1252
DE_DEBTSEC_GDP	10.9900	0.4490	0.4480	0.4390
DK_DEBTSEC_GDP	1.4101	0.2610	0.2580	0.2480
IR_DEBTSEC_GDP	1.1576	0.4220	0.4160	0.3850
IR_HUR	-8.4567	0.0513	0.0493	0.0470
UK_HUR	-22.8039	0.0377	0.0377	0.0370
UK_YIELD10YBY Forecast (1)		**0.5734**	**0.3174**	**0.6108**
UK_YIELD10YBY Real (2)		**0.8330**	**0.4880**	**0.8220**
Difference (2-1)		***0.2596***	***0.1706***	***0.2112***

Table 4.6 - Model B with more recent data vs real figures

Concerning the model B, there were also divergences and it did not show a downward trend along the quarters as the model A. Notwithstanding, the forecasted figures showed the same pace, decrease and an increase.

Both models presented good forecast ability.

5 Conclusion

The Brexit process have been causing some market turbulence and uncertainty in UK and in EU. The studies referred in the Review of Literature chapter confirm the expectation of negative effects (economical, financial, social and political) in the future for some ESMs (including UK) independently how the Brexit process will end.

The level of Sovereign debt among some of ESM has been a concern since the last crisis. This thesis focused on this matter and tried to link with the Brexit's potential effects (a unique event that has never occurred before).

Since all the negative effects are not easy foreseeable, uncertainty have prevailed, although some of them has already been analysed and discussed (despite some imprecisely).

In this dissertation it was used simple procedures, technical standards, and available data to study the impacts of the Brexit on the Sovereign debt. For this reason, it was chosen SBYs, GDP, Total Sovereign Securitized Debt, HUR and two important banking reference rates (in UK and EU) as the main time series used to construct the models in this thesis.

The main goal of this approach was to use the volatility and historic turbulence to find the best and suitable model to foreseen the SBY behaviour. The models were based on the study of the static and dynamic correlation and cointegration between the considered variables.

As referred we started with a significant number of sovereigns in order to try to get a wider "picture" of the European sovereign financial behaviour along the last nineteen years. As several authors stated, each sovereign has its specificities and some of them are more prepared to face financial turbulence and economic crises, meaning that each of them responds differently. It was necessary to separate them in order to reinforce the statistical results. For this reason, it was decided to regroup and use the geographical closeness as one of the main drivers to continue this analysis.

The statistical tests and the models proved the static and dynamic relations between SBYs and macroeconomic variables and regressions were generated in order to reproduce the German and the English SBY for 10 years maturity. Moreover, some of the first concerns about the Brexit consequences (decrease of the GDP, increase of the unemployment rates) on the SBY were statistically proved, using the debt level in percentage of GDP, the unemployment rates as well as the impacts of certain central European SBYs.

Both multiple regression models showed good forecasting abilities by following the same dynamics comparing to the real figures, although some differences were detected.

More tests and other approaches could be used to analyse this event, namely dynamic and nonlinear modelling, as the Autoregressive or Moving Average models, or other more complex as the Autoregressive Conditionally Heteroskedastic (ARCH, and its derivatives) which implies that the variance is not constant along the series).

It also should be referred that the model's residuals may represent a threat in what concerns to modelling in order to get biased results. Since our purpose was to study the static and dynamic correlations, we focused on ensuring the stationarity of first order differences of each SBY so we could study and find cointegration and long-term equilibria between the variables.

Considering all the tests performed and the models created (which were not globally rejected) we can conclude that the Brexit will bring, in a short-term, negative impacts for both sides (caused by the yields variations and/or by the GDPs), namely for the English and the central Europe SBYs. We also know that SBY and sovereign debt reflects, somehow, the economic framework of each country meaning that economic negative impacts are actually expected.

We also believe that more interesting papers, theories and models regarding the Brexit event were published after the research made along the data collect phase and more material could be used in order to enrich this work.

After a very long period of negotiations (still taken in place), United Kingdom left the EU only in 31st January of 2020. It is difficult to detect what have been the consequences of the Brexit accord due to the Pandemic situation, which have been causing a global economic and financial recession.

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7 Annex

7.1 SBY Bloomberg codes

Country	Name	Code
France	France Govt RV Curve 10Y	RV0004P 10Y BLC Curncy
Germany	German Government Bonds 10 Yr	GDBR10 Index
United Kingdom	UK Govt Bonds 10 Year Note Gen	GUKG10 Index
Netherland	NETHERLANDS GOVERNMENT	GTNLG10Y Govt
Denmark	Denmark Government Bonds 10 Ye	GDGB10YR Index
Ireland	Ireland Government Bonds 10 Ye	GIGB10YR Index
France	France Govt RV Curve 5Y	RV0004P 5Y BLC Curncy
Germany	German Government Bonds 5 Yr O	GDBR5 Index
United Kingdom	UK Govt Bonds 5 Year Note Gene	GUKG5 Index
Netherland	NETHERLANDS GOVERNMENT	GTNLG5Y Govt
Denmark	Denmark Government Bonds 5 Yea	GDGB5YR Index
Ireland	Ireland Government Bonds 5 Yea	GIGB5YR Index
France	France Govt RV Curve 2Y	RV0004P 2Y BLC Curncy
Germany	German Government Bonds 2 Yr B	GDBR2 Index
United Kingdom	UK Govt Bonds 2 Year Note Gene	GUKG2 Index
Netherland	NETHERLANDS GOVERNMENT	GTNLG2Y Govt
Denmark	Denmark Government Bonds 2 Yea	GDGB2YR Index
Ireland	Ireland	GIGB2YR Index

Table 7.1 - SBYs Bloomberg codes

7.2 SBY graphs and main stats

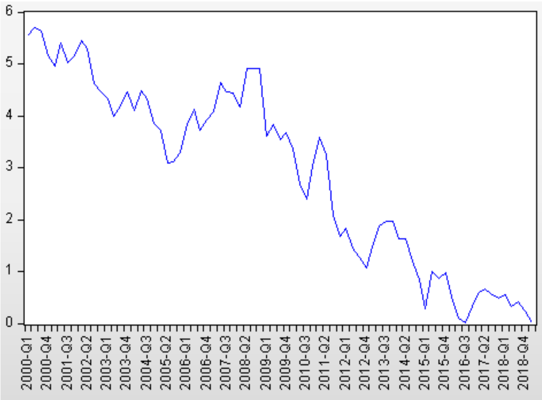


Figure 7.1 - Danish 10 years SBY

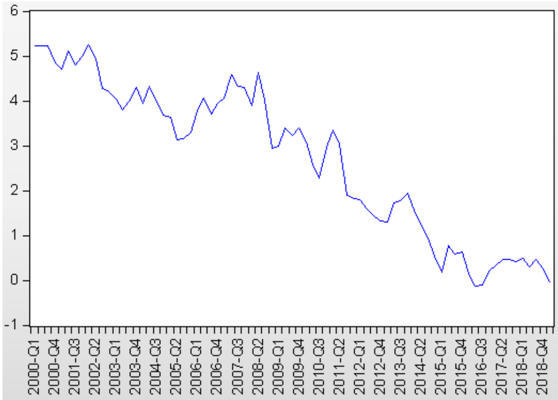


Figure 7.2 - German 10 years SBY

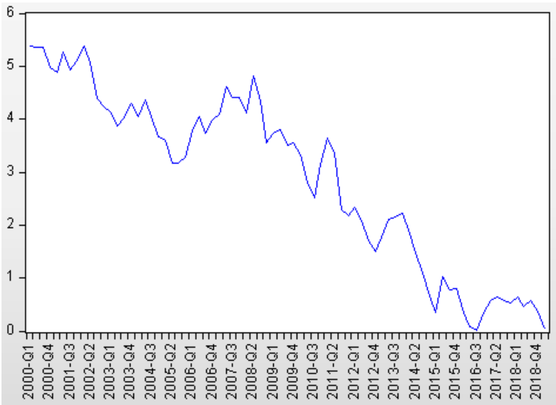


Figure 7.3 - Dutch 10 years SBY

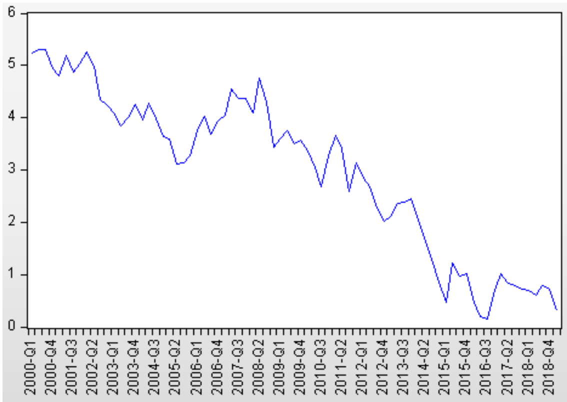


Figure 7.4 - French 10 years SBY

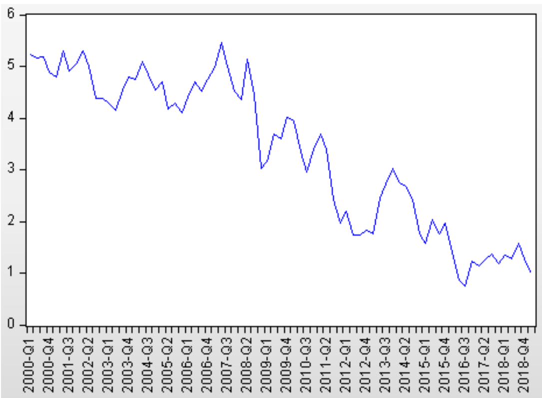


Figure 7.5 - English 10 years SBY

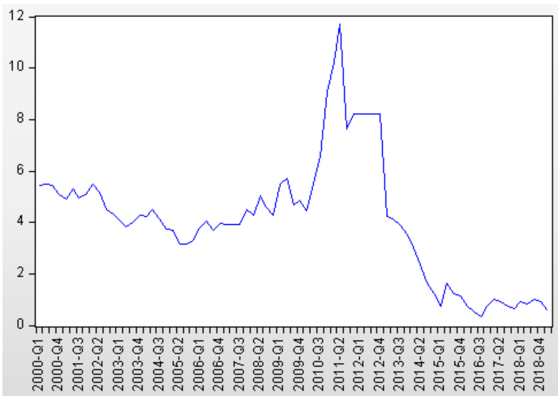
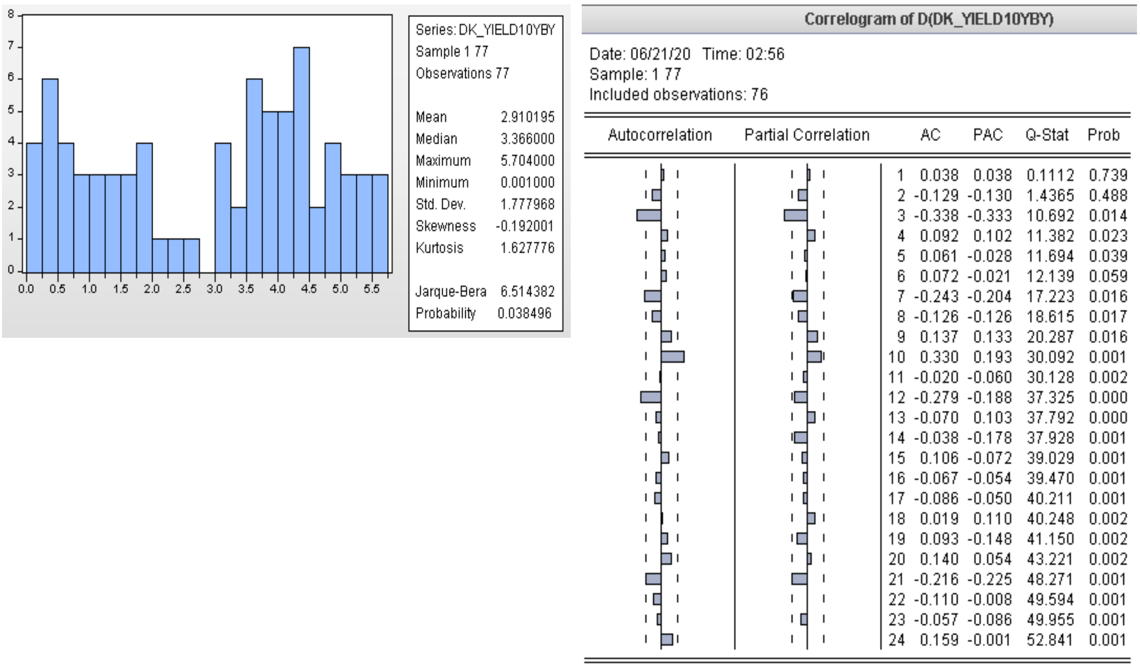
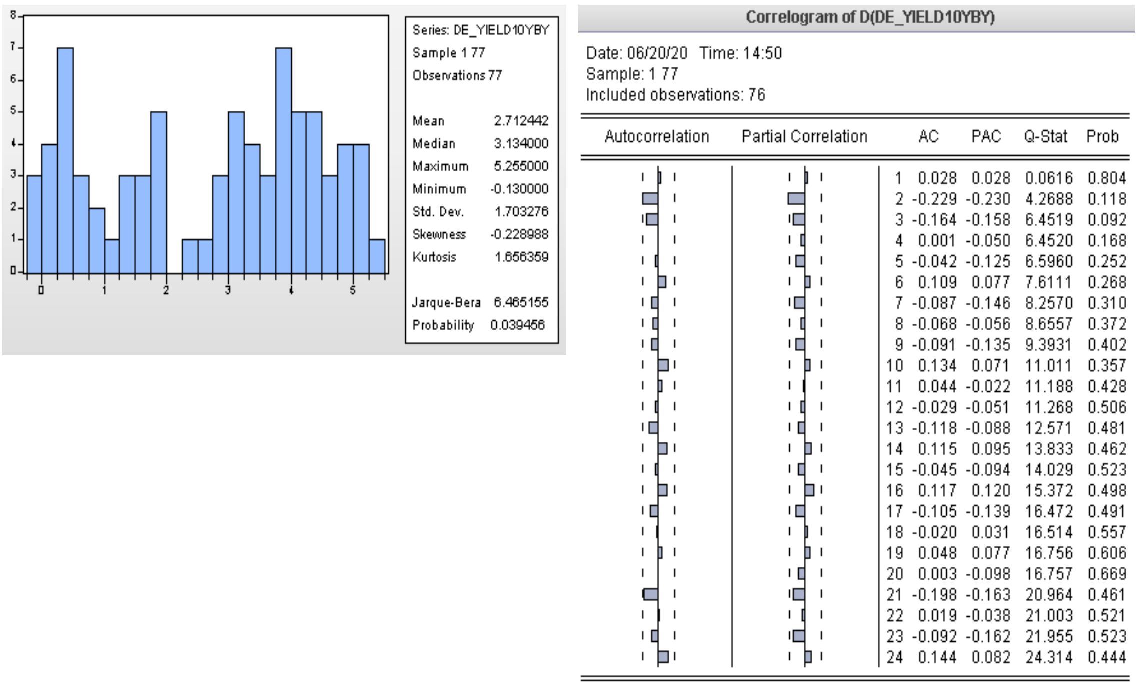


Figure 7.6 - Irish 10 years SBY



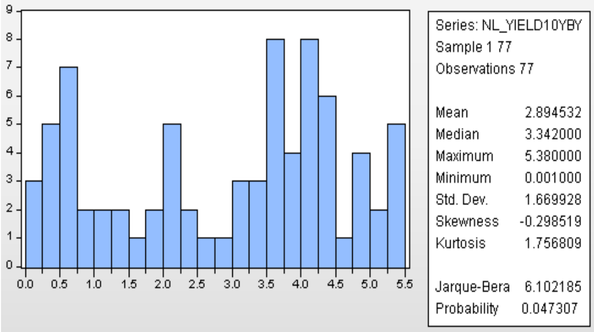
	AC	PAC	Q-Stat	Prob
1	0.038	0.038	0.1112	0.739
2	-0.129	-0.130	1.4365	0.488
3	-0.338	-0.333	10.692	0.014
4	0.092	0.102	11.382	0.023
5	0.061	-0.028	11.694	0.039
6	0.072	-0.021	12.139	0.059
7	-0.243	-0.204	17.223	0.016
8	-0.126	-0.126	18.615	0.017
9	0.137	0.133	20.287	0.016
10	0.330	0.193	30.092	0.001
11	-0.020	-0.060	30.128	0.002
12	-0.279	-0.188	37.325	0.000
13	-0.070	0.103	37.792	0.000
14	-0.038	-0.178	37.928	0.001
15	0.106	-0.072	39.029	0.001
16	-0.067	-0.054	39.470	0.001
17	-0.086	-0.050	40.211	0.001
18	0.019	0.110	40.248	0.002
19	0.093	-0.148	41.150	0.002
20	0.140	0.054	43.221	0.002
21	-0.216	-0.225	48.271	0.001
22	-0.110	-0.008	49.594	0.001
23	-0.057	-0.086	49.955	0.001
24	0.159	-0.001	52.841	0.001

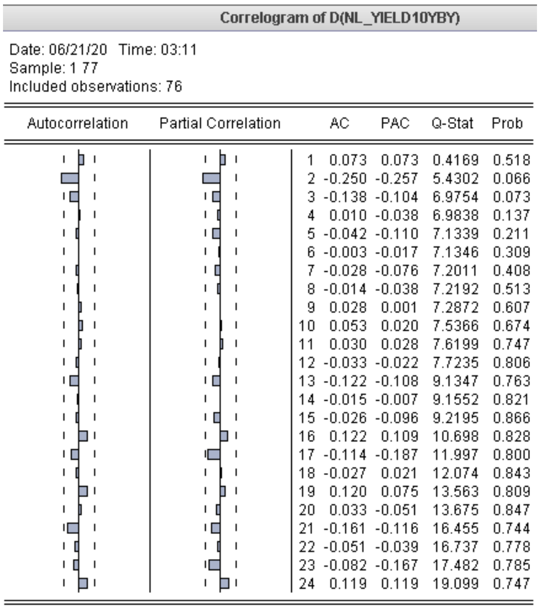
Figure 7.7 - Histrogram, stats and Correlogram (first difference, 24 lags) for the Danish SBY



	AC	PAC	Q-Stat	Prob
1	0.028	0.028	0.0616	0.804
2	-0.229	-0.230	4.2688	0.118
3	-0.164	-0.158	6.4519	0.092
4	0.001	-0.050	6.4520	0.168
5	-0.042	-0.125	6.5960	0.252
6	0.109	0.077	7.6111	0.268
7	-0.087	-0.146	8.2570	0.310
8	-0.068	-0.056	8.6557	0.372
9	-0.091	-0.135	9.3931	0.402
10	0.134	0.071	11.011	0.357
11	0.044	-0.022	11.188	0.428
12	-0.029	-0.051	11.268	0.506
13	-0.118	-0.088	12.571	0.481
14	0.115	0.095	13.833	0.462
15	-0.045	-0.094	14.029	0.523
16	0.117	0.120	15.372	0.498
17	-0.105	-0.139	16.472	0.491
18	-0.020	0.031	16.514	0.557
19	0.048	0.077	16.756	0.606
20	0.003	-0.098	16.757	0.669
21	-0.198	-0.163	20.964	0.461
22	0.019	-0.038	21.003	0.521
23	-0.092	-0.162	21.955	0.523
24	0.144	0.082	24.314	0.444

Figure 7.8 - Histrogram, stats and Correlogram (first difference, 24 lags) for the German SBY



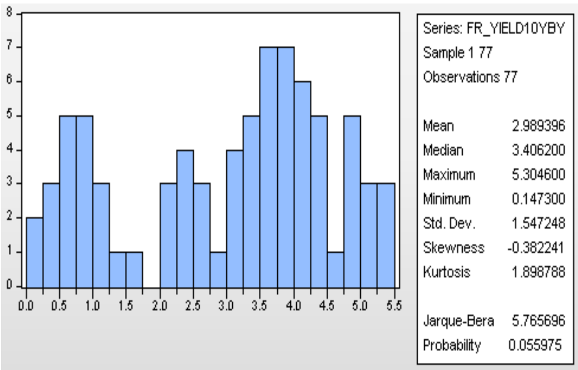


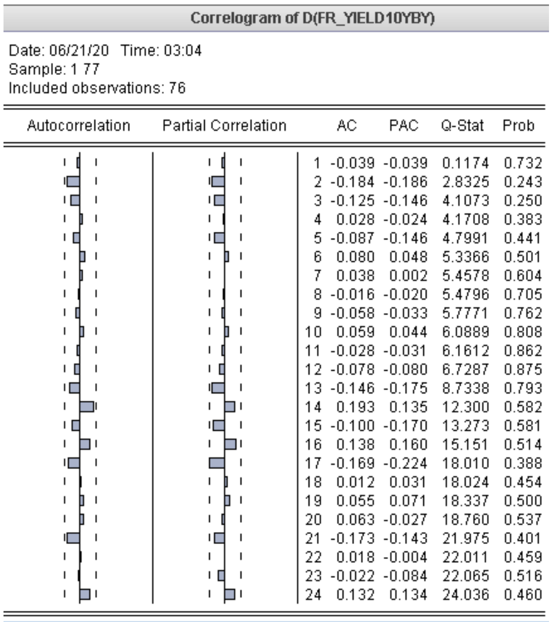
Correlogram of D(NL_YIELD10YBY)

Date: 06/21/20 Time: 03:11
Sample: 1 77
Included observations: 76

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.073	0.073	0.4169	0.518
		2	-0.250	-0.257	5.4302	0.066
		3	-0.138	-0.104	6.9754	0.073
		4	0.010	-0.038	6.9838	0.137
		5	-0.042	-0.110	7.1339	0.211
		6	-0.003	-0.017	7.1346	0.309
		7	-0.028	-0.076	7.2011	0.408
		8	-0.014	-0.038	7.2192	0.513
		9	0.028	0.001	7.2872	0.607
		10	0.053	0.020	7.5366	0.674
		11	0.030	0.028	7.6199	0.747
		12	-0.033	-0.022	7.7235	0.806
		13	-0.122	-0.108	9.1347	0.763
		14	-0.015	-0.007	9.1552	0.821
		15	-0.026	-0.096	9.2195	0.866
		16	0.122	0.109	10.698	0.828
		17	-0.114	-0.187	11.997	0.800
		18	-0.027	0.021	12.074	0.843
		19	0.120	0.075	13.563	0.809
		20	0.033	-0.051	13.675	0.847
		21	-0.161	-0.116	16.455	0.744
		22	-0.051	-0.039	16.737	0.778
		23	-0.082	-0.167	17.482	0.785
		24	0.119	0.119	19.099	0.747

Figure 7.9 - Histrogram, stats and Correlogram (first difference, 24 lags) for the Dutch SBY



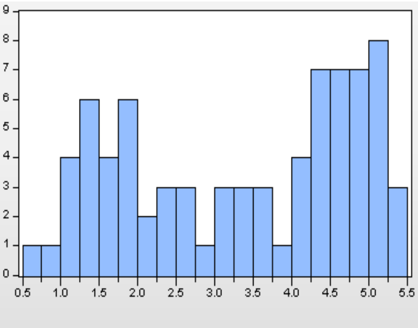


Correlogram of D(FR_YIELD10YBY)

Date: 06/21/20 Time: 03:04
Sample: 1 77
Included observations: 76

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.039	-0.039	0.1174	0.732
		2	-0.184	-0.186	2.8325	0.243
		3	-0.125	-0.146	4.1073	0.250
		4	0.028	-0.024	4.1708	0.383
		5	-0.087	-0.146	4.7991	0.441
		6	0.080	0.048	5.3366	0.501
		7	0.038	0.002	5.4578	0.604
		8	-0.016	-0.020	5.4796	0.705
		9	-0.058	-0.033	5.7771	0.762
		10	0.059	0.044	6.0889	0.808
		11	-0.028	-0.031	6.1612	0.862
		12	-0.078	-0.080	6.7287	0.875
		13	-0.146	-0.175	8.7338	0.793
		14	0.193	0.135	12.300	0.582
		15	-0.100	-0.170	13.273	0.581
		16	0.138	0.160	15.151	0.514
		17	-0.169	-0.224	18.010	0.388
		18	0.012	0.031	18.024	0.454
		19	0.055	0.071	18.337	0.500
		20	0.063	-0.027	18.760	0.537
		21	-0.173	-0.143	21.975	0.401
		22	0.018	-0.004	22.011	0.459
		23	-0.022	-0.084	22.065	0.516
		24	0.132	0.134	24.036	0.460

Figure 7.10 - Histrogram, stats and Correlogram (first difference, 24 lags) for the French SBY



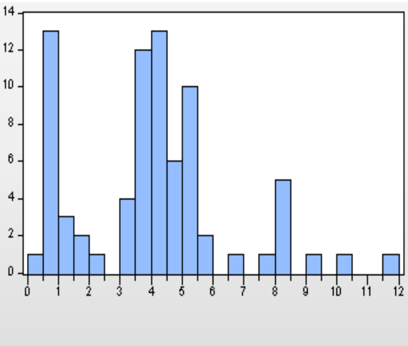
Series: UK_YIELD10YBY	
Sample 1 77	
Observations 77	
Mean	3.360000
Median	3.689000
Maximum	5.463000
Minimum	0.746000
Std. Dev.	1.466729
Skewness	-0.266402
Kurtosis	1.573256
Jarque-Bera	7.441662
Probability	0.024214

Correlogram of D(UK_YIELD10YBY)

Date: 06/21/20 Time: 03:16
Sample: 1 77
Included observations: 76

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.055	0.055	0.2380	0.626
		2	-0.304	-0.308	7.6480	0.022
		3	-0.049	-0.012	7.8468	0.049
		4	0.008	-0.091	7.8515	0.097
		5	-0.044	-0.065	8.0144	0.155
		6	-0.040	-0.066	8.1503	0.227
		7	-0.059	-0.104	8.4501	0.295
		8	-0.151	-0.206	10.441	0.235
		9	-0.089	-0.164	11.149	0.266
		10	0.048	-0.103	11.354	0.331
		11	0.145	0.023	13.264	0.276
		12	0.063	-0.010	13.632	0.325
		13	-0.059	-0.067	13.956	0.377
		14	0.047	0.036	14.171	0.437
		15	0.079	0.011	14.782	0.467
		16	0.034	0.038	14.897	0.532
		17	0.021	0.052	14.943	0.600
		18	-0.092	-0.058	15.812	0.606
		19	0.006	0.110	15.816	0.670
		20	-0.083	-0.091	16.542	0.682
		21	-0.113	-0.069	17.910	0.655
		22	-0.035	-0.090	18.042	0.704
		23	-0.063	-0.155	18.480	0.731
		24	0.189	0.193	22.540	0.547

Figure 7.11 - Histrogram, stats and Correlogram (first difference, 24 lags) for the English SBY



Series: IR_YIELD10YBY	
Sample 1 77	
Observations 77	
Mean	4.060481
Median	4.109000
Maximum	11.69500
Minimum	0.327000
Std. Dev.	2.426168
Skewness	0.589620
Kurtosis	3.527351
Jarque-Bera	5.353759
Probability	0.068777

Correlogram of D(IR_YIELD10YBY)

Date: 06/21/20 Time: 03:08
Sample: 1 77
Included observations: 76

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.000	0.000	3.E-06	0.999
		2	0.068	0.068	0.3696	0.831
		3	-0.111	-0.111	1.3622	0.714
		4	-0.021	-0.025	1.3970	0.845
		5	-0.105	-0.092	2.3235	0.803
		6	0.426	0.428	17.663	0.007
		7	-0.074	-0.107	18.133	0.011
		8	0.074	0.011	18.614	0.017
		9	-0.239	-0.212	23.680	0.005
		10	-0.116	-0.103	24.888	0.006
		11	-0.026	0.123	24.948	0.009
		12	0.129	-0.101	26.486	0.009
		13	-0.104	-0.068	27.501	0.011
		14	0.089	-0.004	28.250	0.013
		15	-0.270	-0.137	35.336	0.002
		16	-0.079	0.021	35.958	0.003
		17	0.006	-0.002	35.962	0.005
		18	0.108	0.047	37.155	0.005
		19	-0.036	-0.035	37.293	0.007
		20	0.054	-0.065	37.604	0.010
		21	-0.150	0.055	40.035	0.007
		22	-0.097	-0.156	41.063	0.008
		23	-0.064	-0.048	41.524	0.010
		24	0.013	-0.113	41.544	0.015

Figure 7.12 - Histrogram, stats and Correlogram (first difference, 24 lags) for the Irish SBY

	DK_YIELD 10YBY	DE_YIELD 10YBY	NL_YIELD 10YBY	FR_YIELD 10YBY	UK_YIELD 10YBY	IR_YIELD 10YBY
Mean	2.910	2.712	2.895	2.989	3.360	4.060
Median	3.366	3.134	3.342	3.406	3.689	4.109
Maximum	5.704	5.255	5.380	5.305	5.463	11.695
Minimum	0.001	-0.130	0.001	0.147	0.746	0.327
Std. Dev.	1.778	1.703	1.670	1.547	1.467	2.426
Skewness	-0.192	-0.229	-0.299	-0.382	-0.266	0.590
Kurtosis	1.628	1.656	1.757	1.899	1.573	3.527
Jarque-Bera	6.514	6.465	6.102	5.766	7.442	5.354
Probability	0.038	0.039	0.047	0.056	0.024	0.069
Sum	224.085	208.858	222.879	230.184	258.720	312.657
Sum Sq. Dev.	240.249	220.487	211.938	181.942	163.498	447.358
Observations	77	77	77	77	77	77

Table 7.2 - SBYs General Stats (summary)

7.3 Vector Error Correction Model

Lag selection criteria for 10 years SBY.

VAR Lag Order Selection Criteria
Endogenous variables: UK_YIELD10YBY NL_YIELD10YBY IR_YIELD10YBY FR_YIE...
Exogenous variables: C
Date: 06/08/20 Time: 01:25
Sample: 1 77
Included observations: 70

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-169.4224	NA	6.05e-06	5.012068	5.204796	5.088622
1	111.2532	505.2160	5.59e-09	-1.978662	-0.629565*	-1.442784*
2	142.3753	50.68467	6.57e-09	-1.839295	0.666171	-0.844093
3	175.5403	48.32610	7.55e-09	-1.758294	1.903541	-0.303769
4	217.3643	53.77374	7.16e-09	-1.924695	2.893509	-0.010845
5	278.7487	68.39974	4.23e-09	-2.649963	3.324610	-0.276790
6	339.2966	57.08801	2.89e-09	-3.351331	3.779611	-0.518834
7	408.0742	53.05701*	1.87e-09*	-4.287834*	3.999477	-0.996013

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Figure 7.13 - VAR lag order selection criteria for the SBY

VECM outputs for the English SBY as the target variable and German, Irish, Danish, Dutch and French SBY.

Vector Error Correction Estimates

Vector Error Correction Estimates
Date: 11/23/19 Time: 01:15
Sample (adjusted): 4 77
Included observations: 74 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
UK_YIELD10YBY(-1)	1.000000
NL_YIELD10YBY(-1)	-29.46848
	(3.50293)
	[-8.41253]
IR_YIELD10YBY(-1)	0.220338
	(0.10790)
	[2.04213]
FR_YIELD10YBY(-1)	13.98361
	(1.78053)
	[7.85361]
DK_YIELD10YBY(-1)	10.33582
	(1.19794)
	[8.62803]
DE_YIELD10YBY(-1)	5.753128
	(1.41089)
	[4.07765]
@TREND(1)	0.090244
	(0.02074)
	[4.35084]
C	-9.936278

Error Correction:	D(UK_YIEL...	D(NL_YIELD...	D(IR_YIELD...	D(FR_YIELD...	D(DK_YIEL...	D(DE_YIEL...
CointEq1	-0.123031	-0.087803	-0.132899	-0.122975	-0.135593	-0.116482
	(0.04948)	(0.04426)	(0.11699)	(0.04082)	(0.04833)	(0.04449)
	[-2.48658]	[-1.98369]	[-1.13599]	[-3.01273]	[-2.80555]	[-2.61799]
D(UK_YIELD10YBY(-1))	-0.325125	-0.174719	-0.837212	0.011124	-0.141021	-0.185852
	(0.28935)	(0.25885)	(0.68416)	(0.23871)	(0.28264)	(0.26020)
	[-1.12364]	[-0.67499]	[-1.22371]	[0.04660]	[-0.49895]	[-0.71427]
D(UK_YIELD10YBY(-2))	-0.671795	-0.640445	-0.632536	-0.716725	-0.453742	-0.620084
	(0.29606)	(0.26485)	(0.70002)	(0.24424)	(0.28919)	(0.26623)
	[-2.26915]	[-2.41817]	[-0.90360]	[-2.93450]	[-1.56902]	[-2.32914]
D(NL_YIELD10YBY(-1))	-1.213119	-0.766315	-0.621973	-1.524510	-1.332167	-0.946475
	(1.07876)	(0.96505)	(2.55070)	(0.88996)	(1.05374)	(0.97008)
	[-1.12455]	[-0.79407]	[-0.24384]	[-1.71301]	[-1.26423]	[-0.97567]
D(NL_YIELD10YBY(-2))	-0.890552	-1.110678	-1.222742	-1.037531	-1.419634	-0.935589
	(0.71941)	(0.64357)	(1.70102)	(0.59350)	(0.70272)	(0.64693)
	[-1.23790]	[-1.72580]	[-0.71883]	[-1.74817]	[-2.02020]	[-1.44621]
D(IR_YIELD10YBY(-1))	-0.050393	-0.021882	0.001126	-0.055813	-0.004716	-0.038681
	(0.06966)	(0.06232)	(0.16471)	(0.05747)	(0.06805)	(0.06264)
	[-0.72340]	[-0.35114]	[0.00683]	[-0.97118]	[-0.06931]	[-0.61749]
D(IR_YIELD10YBY(-2))	-0.069791	-0.026271	0.124554	0.051920	-0.035977	-0.011858
	(0.06681)	(0.05977)	(0.15796)	(0.05512)	(0.06526)	(0.06008)
	[-1.04465]	[-0.43957]	[0.78849]	[0.94202]	[-0.55131]	[-0.19738]
D(FR_YIELD10YBY(-1))	0.746286	0.801434	1.423924	1.151297	1.265325	0.870043
	(0.63133)	(0.56478)	(1.49276)	(0.52083)	(0.61668)	(0.56772)
	[1.18209]	[1.41903]	[0.95389]	[2.21049]	[2.05182]	[1.53252]
D(FR_YIELD10YBY(-2))	0.218123	0.269299	1.503506	0.438913	0.382330	0.316840
	(0.51714)	(0.46263)	(1.22277)	(0.42663)	(0.50515)	(0.46504)
	[0.42179]	[0.58211]	[1.22960]	[1.02879]	[0.75687]	[0.68132]
D(DK_YIELD10YBY(-1))	0.523447	0.535889	1.009474	0.788094	0.491044	0.544240
	(0.44360)	(0.39683)	(1.04887)	(0.36596)	(0.43331)	(0.39890)
	[1.18001]	[1.35041]	[0.96244]	[2.15351]	[1.13325]	[1.36434]
D(DK_YIELD10YBY(-2))	0.242590	0.148230	0.395475	0.340931	0.179329	0.185573
	(0.31074)	(0.27798)	(0.73473)	(0.25635)	(0.30353)	(0.27943)
	[0.78069]	[0.53324]	[0.53826]	[1.32993]	[0.59081]	[0.66411]
D(DE_YIELD10YBY(-1))	0.403658	-0.258704	-0.882127	-0.306076	0.010452	-0.164035
	(0.71124)	(0.63627)	(1.68171)	(0.58676)	(0.69474)	(0.63958)
	[0.56754]	[-0.40660]	[-0.52454]	[-0.52164]	[0.01504]	[-0.25647]
D(DE_YIELD10YBY(-2))	0.889174	1.068736	-0.453752	0.664635	1.231380	0.794286
	(0.70984)	(0.63502)	(1.67840)	(0.58561)	(0.69338)	(0.63833)
	[1.25264]	[1.68301]	[-0.27035]	[1.13495]	[1.77591]	[1.24433]
C	-0.054436	-0.072578	-0.062635	-0.070528	-0.060164	-0.073394
	(0.04476)	(0.04004)	(0.10582)	(0.03692)	(0.04372)	(0.04025)
	[-1.21630]	[-1.81277]	[-0.59189]	[-1.91019]	[-1.37622]	[-1.82363]

Figure 7.14 - Vector Error Correction estimates for the UK SBY

VEC Residual Serial Correlation LM Tests
Date: 11/23/19 Time: 01:18
Sample: 1 77
Included observations: 74

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	30.15678	36	0.7422	0.829150	(36, 217.9)	0.7444
2	34.63024	36	0.5337	0.961465	(36, 217.9)	0.5368

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	30.15678	36	0.7422	0.829150	(36, 217.9)	0.7444
2	67.73604	72	0.6205	0.930984	(72, 239.7)	0.6327

Figure 7.15 - VEC Residual Serial Correlation LM tests for UK SBY

VEC Residual Heteroskedasticity Tests (Levels and Squares)
Date: 11/23/19 Time: 01:20
Sample: 1 77
Included observations: 74

Joint test:

Chi-sq	df	Prob.
647.3165	546	0.0018

Figure 7.16 - VEC Residual Heteroscedasticity tests for UK SBY

VEC Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: Residuals are multivariate normal
Date: 11/23/19 Time: 01:19
Sample: 1 77
Included observations: 74

Component	Jarque-Bera	df	Prob.
1	1.693782	2	0.4287
2	3.758613	2	0.1527
3	660.1070	2	0.0000
4	0.610057	2	0.7371
5	78.61033	2	0.0000
6	0.223117	2	0.8944
Joint	745.0029	12	0.0000

Figure 7.17 - VEC Residual Normality tests for UK SBY

Dependent Variable: D(UK_YIELD10YBY)
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 06/11/20 Time: 05:26
Sample (adjusted): 4 77
Included observations: 74 after adjustments
D(UK_YIELD10YBY) = C(1)*(UK_YIELD10YBY(-1) - 29.4684835654
*NL_YIELD10YBY(-1) + 0.220338194262*IR_YIELD10YBY(-1) +
13.9836132906*FR_YIELD10YBY(-1) + 10.3358215098
*DK_YIELD10YBY(-1) + 5.75312764008*DE_YIELD10YBY(-1) +
0.0902443716281*@TREND(1) - 9.93627753972) + C(2)
*D(UK_YIELD10YBY(-1)) + C(3)*D(UK_YIELD10YBY(-2)) + C(4)
*D(NL_YIELD10YBY(-1)) + C(5)*D(NL_YIELD10YBY(-2)) + C(6)
*D(IR_YIELD10YBY(-1)) + C(7)*D(IR_YIELD10YBY(-2)) + C(8)
*D(FR_YIELD10YBY(-1)) + C(9)*D(FR_YIELD10YBY(-2)) + C(10)
*D(DK_YIELD10YBY(-1)) + C(11)*D(DK_YIELD10YBY(-2)) + C(12)
*D(DE_YIELD10YBY(-1)) + C(13)*D(DE_YIELD10YBY(-2)) + C(14)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.123031	0.049478	-2.486579	0.0157
C(2)	-0.325125	0.289349	-1.123645	0.2656
C(3)	-0.671795	0.296056	-2.269152	0.0269
C(4)	-1.213119	1.078761	-1.124548	0.2653
C(5)	-0.890552	0.719406	-1.237899	0.2206
C(6)	-0.050393	0.069661	-0.723399	0.4722
C(7)	-0.069791	0.066808	-1.044651	0.3004
C(8)	0.746286	0.631327	1.182091	0.2418
C(9)	0.218123	0.517141	0.421786	0.6747
C(10)	0.523447	0.443596	1.180007	0.2427
C(11)	0.242590	0.310737	0.780691	0.4381
C(12)	0.403658	0.711240	0.567541	0.5725
C(13)	0.889174	0.709843	1.252636	0.2152
C(14)	-0.054436	0.044755	-1.216304	0.2286

Figure 7.18 - OLS equation for the UK SBY VECM vector

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.048380	Prob. F(2,58)	0.9528
Obs*R-squared	0.123248	Prob. Chi-Square(2)	0.9402

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 06/11/20 Time: 07:18
Sample: 4 77
Included observations: 74
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.007224	0.055391	-0.130421	0.8967
C(2)	-0.064169	0.397426	-0.161463	0.8723
C(3)	-0.058808	0.387890	-0.151611	0.8800
C(4)	-0.199972	1.272836	-0.157107	0.8757
C(5)	-0.076343	0.850029	-0.089812	0.9287
C(6)	0.000131	0.071421	0.001840	0.9985
C(7)	-0.002682	0.070517	-0.038040	0.9698
C(8)	0.121071	0.750416	0.161338	0.8724
C(9)	0.054500	0.586756	0.092883	0.9263
C(10)	0.071279	0.506302	0.140784	0.8885
C(11)	0.018796	0.361349	0.052016	0.9587
C(12)	0.021523	0.729175	0.029517	0.9766
C(13)	0.038699	0.734407	0.052694	0.9582
C(14)	-0.003859	0.047169	-0.081819	0.9351
RESID(-1)	0.081966	0.335143	0.244569	0.8077
RESID(-2)	0.043968	0.310600	0.141559	0.8879

Figure 7.19 - Residual Serial Correlation LM Test for the UK SBY VECM vector

VECM for the German SBY as the target variable and English, Irish, Danish, Dutch and French SBY.

Vector Error Correction Estimates

Vector Error Correction Estimates
Date: 06/03/20 Time: 02:08
Sample (adjusted): 4 77
Included observations: 74 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
DE_YIELD10YBY(-1)	1.000000
NL_YIELD10YBY(-1)	-5.122168 (0.50183) [-10.2069]
IR_YIELD10YBY(-1)	0.038299 (0.01761) [2.17449]
FR_YIELD10YBY(-1)	2.430611 (0.32812) [7.40760]
DK_YIELD10YBY(-1)	1.796557 (0.20147) [8.91733]
UK_YIELD10YBY(-1)	0.173818 (0.07592) [2.28962]
@TREND(1)	0.015686 (0.00292) [5.37522]
C	-1.727109

Error Correction:	D(DE_YIEL...	D(NL_YIELD...	D(IR_YIELD...	D(FR_YIELD...	D(DK_YIEL...	D(UK_YIEL...
CointEq1	-0.670139 (0.25597) [-2.61799]	-0.505140 (0.25465) [-1.98369]	-0.764585 (0.67305) [-1.13599]	-0.707489 (0.23483) [-3.01273]	-0.780084 (0.27805) [-2.80555]	-0.707812 (0.28465) [-2.48658]
D(DE_YIELD10YBY(-1))	-0.164035 (0.63958) [-0.25647]	-0.258704 (0.63627) [-0.40660]	-0.882127 (1.68171) [-0.52454]	-0.306076 (0.58676) [-0.52164]	0.010452 (0.69474) [0.01504]	0.403658 (0.71124) [0.56754]
D(DE_YIELD10YBY(-2))	0.794286 (0.63833) [1.24433]	1.068736 (0.63502) [1.68301]	-0.453752 (1.67840) [-0.27035]	0.664635 (0.58561) [1.13495]	1.231380 (0.69338) [1.77591]	0.889174 (0.70984) [1.25264]
D(NL_YIELD10YBY(-1))	-0.946475 (0.97008) [-0.97567]	-0.766315 (0.96505) [-0.79407]	-0.621973 (2.55070) [-0.24384]	-1.524510 (0.88996) [-1.71301]	-1.332167 (1.05374) [-1.26423]	-1.213119 (1.07876) [-1.12455]
D(NL_YIELD10YBY(-2))	-0.935589 (0.64693) [-1.44621]	-1.110678 (0.64357) [-1.72580]	-1.222742 (1.70102) [-0.71883]	-1.037531 (0.59350) [-1.74817]	-1.419634 (0.70272) [-2.02020]	-0.890552 (0.71941) [-1.23790]
D(IR_YIELD10YBY(-1))	-0.038681 (0.06264) [-0.61749]	-0.021882 (0.06232) [-0.35114]	0.001126 (0.16471) [0.00683]	-0.055813 (0.05747) [-0.97118]	-0.004716 (0.06805) [-0.06931]	-0.050393 (0.06966) [-0.72340]
D(IR_YIELD10YBY(-2))	-0.011858 (0.06008) [-0.19738]	-0.026271 (0.05977) [-0.43957]	0.124554 (0.15796) [0.78849]	0.051920 (0.05512) [0.94202]	-0.035977 (0.06526) [-0.55131]	-0.069791 (0.06681) [-1.04465]
D(FR_YIELD10YBY(-1))	0.870043 (0.56772) [1.53252]	0.801434 (0.56478) [1.41903]	1.423924 (1.49276) [0.95389]	1.151297 (0.52083) [2.21049]	1.265325 (0.61668) [2.05182]	0.746286 (0.63133) [1.18209]
D(FR_YIELD10YBY(-2))	0.316840 (0.46504) [0.68132]	0.269299 (0.46263) [0.58211]	1.503506 (1.22277) [1.22960]	0.438913 (0.42663) [1.02879]	0.382330 (0.50515) [0.75687]	0.218123 (0.51714) [0.42179]
D(DK_YIELD10YBY(-1))	0.544240 (0.39890) [1.36434]	0.535889 (0.39683) [1.35041]	1.009474 (1.04887) [0.96244]	0.788094 (0.36596) [2.15351]	0.491044 (0.43331) [1.13325]	0.523447 (0.44360) [1.18001]
D(DK_YIELD10YBY(-2))	0.185573 (0.27943) [0.66411]	0.148230 (0.27798) [0.53324]	0.395475 (0.73473) [0.53826]	0.340931 (0.25635) [1.32993]	0.179329 (0.30353) [0.59081]	0.242590 (0.31074) [0.78069]
D(UK_YIELD10YBY(-1))	-0.185852 (0.26020) [-0.71427]	-0.174719 (0.25885) [-0.67499]	-0.837212 (0.68416) [-1.22371]	0.011124 (0.23871) [0.04660]	-0.141021 (0.28264) [-0.49895]	-0.325125 (0.28935) [-1.12364]
D(UK_YIELD10YBY(-2))	-0.620084 (0.26623) [-2.32914]	-0.640445 (0.26485) [-2.41817]	-0.632536 (0.70002) [-0.90360]	-0.716725 (0.24424) [-2.93450]	-0.453742 (0.28919) [-1.56902]	-0.671795 (0.29606) [-2.26915]
C	-0.073394 (0.04025) [-1.82363]	-0.072578 (0.04004) [-1.81277]	-0.062635 (0.10582) [-0.59189]	-0.070528 (0.03692) [-1.91019]	-0.060164 (0.04372) [-1.37622]	-0.054436 (0.04476) [-1.21630]

Figure 7.20 - Vector Error Correction estimates for the DE SBY

VEC Residual Serial Correlation LM Tests
Date: 07/03/20 Time: 02:50
Sample: 1 77
Included observations: 74

Null hypothesis: No serial correlation at lag h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	30.15678	36	0.7422	0.829150	(36, 217.9)	0.7444
2	34.63024	36	0.5337	0.961465	(36, 217.9)	0.5368

Null hypothesis: No serial correlation at lags 1 to h

Lag	LRE* stat	df	Prob.	Rao F-stat	df	Prob.
1	30.15678	36	0.7422	0.829150	(36, 217.9)	0.7444
2	67.73604	72	0.6205	0.930984	(72, 239.7)	0.6327

*Edgeworth expansion corrected likelihood ratio statistic.

Figure 7.21 - VEC Residual Serial Correlation LM tests for DE SBY

VEC Residual Heteroskedasticity Tests (Levels and Squares)
Date: 06/03/20 Time: 02:31
Sample: 1 77
Included observations: 74

Joint test:

Chi-sq	df	Prob.
647.3165	546	0.0018

Figure 7.22 - VEC Residual Heteroscedasticity tests for DE SBY

VEC Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
Null Hypothesis: Residuals are multivariate normal
Date: 06/03/20 Time: 02:28
Sample: 1 77
Included observations: 74

Component	Jarque-Bera	df	Prob.
1	0.047269	2	0.9766
2	4.641433	2	0.0982
3	219.3385	2	0.0000
4	0.721701	2	0.6971
5	93.81366	2	0.0000
6	9.903474	2	0.0071
Joint	328.4660	12	0.0000

Figure 7.23 - VEC Residual Normality tests for DE SBY

Dependent Variable: D(DE_YIELD10YBY)
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 06/11/20 Time: 06:02
Sample (adjusted): 4 77
Included observations: 74 after adjustments
D(DE_YIELD10YBY) = C(1)*(DE_YIELD10YBY(-1) - 5.12216752502
*NL_YIELD10YBY(-1) + 0.0382988537795*IR_YIELD10YBY(-1) +
2.43061064613*FR_YIELD10YBY(-1) + 1.79655696107
*DK_YIELD10YBY(-1) + 0.173818497095*UK_YIELD10YBY(-1) +
0.0156861410477*@TREND(1) - 1.72710882868) + C(2)
*D(DE_YIELD10YBY(-1)) + C(3)*D(DE_YIELD10YBY(-2)) + C(4)
*D(NL_YIELD10YBY(-1)) + C(5)*D(NL_YIELD10YBY(-2)) + C(6)
*D(IR_YIELD10YBY(-1)) + C(7)*D(IR_YIELD10YBY(-2)) + C(8)
*D(FR_YIELD10YBY(-1)) + C(9)*D(FR_YIELD10YBY(-2)) + C(10)
*D(DK_YIELD10YBY(-1)) + C(11)*D(DK_YIELD10YBY(-2)) + C(12)
*D(UK_YIELD10YBY(-1)) + C(13)*D(UK_YIELD10YBY(-2)) + C(14)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.670139	0.255974	-2.617991	0.0112
C(2)	-0.164035	0.639584	-0.256471	0.7985
C(3)	0.794286	0.638327	1.244325	0.2182
C(4)	-0.946475	0.970077	-0.975670	0.3331
C(5)	-0.935589	0.646927	-1.446205	0.1533
C(6)	-0.038681	0.062643	-0.617491	0.5392
C(7)	-0.011858	0.060077	-0.197380	0.8442
C(8)	0.870043	0.567722	1.532517	0.1307
C(9)	0.316840	0.465039	0.681319	0.4983
C(10)	0.544240	0.398904	1.364339	0.1776
C(11)	0.185573	0.279431	0.664112	0.5092
C(12)	-0.185852	0.260197	-0.714275	0.4778
C(13)	-0.620084	0.266228	-2.329142	0.0232
C(14)	-0.073394	0.040246	-1.823630	0.0732

Figure 7.24 - OLS equation for the DE SBY VECM vector

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.134386	Prob. F(2,58)	0.8745
Obs*R-squared	0.341335	Prob. Chi-Square(2)	0.8431

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 06/11/20 Time: 07:01
Sample: 4 77
Included observations: 74
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.039461	0.283290	-0.139294	0.8897
C(2)	0.021845	0.714772	0.030562	0.9757
C(3)	-0.129623	0.695706	-0.186318	0.8528
C(4)	-0.248666	1.169224	-0.212676	0.8323
C(5)	-0.192045	0.769860	-0.249455	0.8039
C(6)	-0.003840	0.064366	-0.059660	0.9526
C(7)	0.004008	0.062576	0.064048	0.9492
C(8)	0.138626	0.697588	0.198722	0.8432
C(9)	0.141936	0.545569	0.260161	0.7957
C(10)	0.078703	0.452348	0.173987	0.8625
C(11)	0.067078	0.314759	0.213108	0.8320
C(12)	0.009047	0.267901	0.033768	0.9732
C(13)	0.007149	0.279068	0.025617	0.9797
C(14)	-0.008360	0.045372	-0.184253	0.8545
RESID(-1)	0.031788	0.329558	0.096458	0.9235
RESID(-2)	0.152291	0.308612	0.493472	0.6235

Figure 7.25 - Residual Serial Correlation LM Test for the DE SBY VECM vector

7.4 Granger Causality between Macroeconomic and SBY variables

Pairwise Granger Causality Tests macroeconomic variables (Debt securities as % of GDP, Harmonized Unemployment Rate and the Reference rate) and all SBY and 2 lags (excluding Granger Causality between the SBY).

Null Hypothesis:	Obs	F-Statistic	Prob.
DE_DEBTSEC_GDP does not Granger Cause DE_HUR	74	3.7246	0.0291
DE_DEBTSEC_GDP does not Granger Cause DK_HUR	74	3.99206	0.0229
DE_DEBTSEC_GDP does not Granger Cause FR_HUR	74	3.31803	0.0421
DE_DEBTSEC_GDP does not Granger Cause IR_DEBTSEC_GDP	74	5.01925	0.0092
DE_DEBTSEC_GDP does not Granger Cause NL_DEBTSEC_GDP	74	5.31958	0.0071
DE_DEBTSEC_GDP does not Granger Cause NL_HUR	74	2.62979	0.0793
DE_DEBTSEC_GDP does not Granger Cause UK_DEBTSEC_GDP	74	7.3772	0.0012
DE_HUR does not Granger Cause DE_DEBTSEC_GDP	74	4.2628	0.0180
DE_HUR does not Granger Cause DE_YIELD10YBY	75	2.6911	0.0748
DE_HUR does not Granger Cause DK_YIELD10YBY	75	2.97894	0.0573
DE_HUR does not Granger Cause EURR002W_INDEX	75	5.1201	0.0084
DE_HUR does not Granger Cause FR_YIELD10YBY	75	2.91975	0.0605
DE_HUR does not Granger Cause NL_YIELD10YBY	75	2.76979	0.0696
DE_HUR does not Granger Cause UK_YIELD10YBY	75	5.06092	0.0089
DE_HUR does not Granger Cause UKBRBASE_INDEX	75	3.29628	0.0428
DE_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	74	2.79478	0.0681
DE_YIELD10YBY does not Granger Cause DE_HUR	75	6.0941	0.0036
DE_YIELD10YBY does not Granger Cause EURR002W_INDEX	75	7.3647	0.0013
DE_YIELD10YBY does not Granger Cause FR_HUR	75	3.34238	0.0411
DE_YIELD10YBY does not Granger Cause IR_HUR	75	3.40388	0.0388
DE_YIELD10YBY does not Granger Cause UKBRBASE_INDEX	75	3.49371	0.0358
DK_DEBTSEC_GDP does not Granger Cause DK_HUR	74	3.30411	0.0426
DK_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	74	7.42266	0.0012
DK_DEBTSEC_GDP does not Granger Cause FR_HUR	74	4.36443	0.0164
DK_DEBTSEC_GDP does not Granger Cause IR_DEBTSEC_GDP	74	2.54638	0.0857
DK_DEBTSEC_GDP does not Granger Cause IR_HUR	74	5.9051	0.0043
DK_DEBTSEC_GDP does not Granger Cause UK_DEBTSEC_GDP	74	7.64513	0.0010
DK_HUR does not Granger Cause DE_DEBTSEC_GDP	74	3.97546	0.0232
DK_HUR does not Granger Cause DK_DEBTSEC_GDP	74	3.52468	0.0349
DK_HUR does not Granger Cause FR_DEBTSEC_GDP	74	3.24765	0.0449
DK_HUR does not Granger Cause FR_HUR	75	2.75029	0.0708
DK_HUR does not Granger Cause IR_HUR	75	11.1845	0.0001
DK_HUR does not Granger Cause NL_DEBTSEC_GDP	74	4.53271	0.0141
DK_HUR does not Granger Cause UK_DEBTSEC_GDP	74	10.7499	0.0001
DK_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	74	6.21361	0.0033
DK_YIELD10YBY does not Granger Cause DE_HUR	75	5.50797	0.0060
DK_YIELD10YBY does not Granger Cause IR_HUR	75	3.75038	0.0284
EURR002W_INDEX does not Granger Cause DE_DEBTSEC_GDP	74	15.8051	0.0000
EURR002W_INDEX does not Granger Cause DE_HUR	75	6.67965	0.0022
EURR002W_INDEX does not Granger Cause DK_HUR	75	2.72333	0.0726
EURR002W_INDEX does not Granger Cause DK_YIELD10YBY	75	4.44236	0.0153
EURR002W_INDEX does not Granger Cause FR_DEBTSEC_GDP	74	8.22693	0.0006
EURR002W_INDEX does not Granger Cause FR_HUR	75	6.02696	0.0038
EURR002W_INDEX does not Granger Cause IR_HUR	75	7.63831	0.0010
EURR002W_INDEX does not Granger Cause NL_YIELD10YBY	75	3.21521	0.0461
EURR002W_INDEX does not Granger Cause UK_HUR	74	3.66423	0.0307
FR_DEBTSEC_GDP does not Granger Cause DE_DEBTSEC_GDP	74	9.48449	0.0002
FR_DEBTSEC_GDP does not Granger Cause DE_HUR	74	6.17757	0.0034

Figure 7.26 - Pairwise Granger Causality tests between the SBY and Macroeconomic variables (2 lags) (1/4)

FR_DEBTSEC_GDP does not Granger Cause DE_YIELD10YBY	74	6.30964	0.0030
FR_DEBTSEC_GDP does not Granger Cause DK_DEBTSEC_GDP	74	6.05432	0.0038
FR_DEBTSEC_GDP does not Granger Cause DK_HUR	74	5.98343	0.0040
FR_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	74	8.64714	0.0004
FR_DEBTSEC_GDP does not Granger Cause EURR002W_INDEX	74	2.58307	0.0828
FR_DEBTSEC_GDP does not Granger Cause FR_YIELD10YBY	74	4.14506	0.0200
FR_DEBTSEC_GDP does not Granger Cause NL_YIELD10YBY	74	6.42735	0.0028
FR_DEBTSEC_GDP does not Granger Cause UK_DEBTSEC_GDP	74	9.51662	0.0002
FR_DEBTSEC_GDP does not Granger Cause UK_YIELD10YBY	74	7.22051	0.0014
FR_HUR does not Granger Cause DE_DEBTSEC_GDP	74	16.5385	0.0000
FR_HUR does not Granger Cause FR_DEBTSEC_GDP	74	2.90156	0.0617
FR_HUR does not Granger Cause IR_DEBTSEC_GDP	74	7.61216	0.0010
FR_HUR does not Granger Cause IR_HUR	75	16.3935	0.0000
FR_HUR does not Granger Cause NL_DEBTSEC_GDP	74	3.47763	0.0364
FR_HUR does not Granger Cause NL_HUR	75	5.49078	0.0061
FR_HUR does not Granger Cause UK_DEBTSEC_GDP	74	12.2811	0.0000
FR_HUR does not Granger Cause UK_HUR	74	8.1741	0.0007
FR_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	74	3.48738	0.0361
FR_YIELD10YBY does not Granger Cause DE_HUR	75	5.34163	0.0069
FR_YIELD10YBY does not Granger Cause EURR002W_INDEX	75	4.74383	0.0117
FR_YIELD10YBY does not Granger Cause FR_HUR	75	3.21851	0.0460
FR_YIELD10YBY does not Granger Cause IR_HUR	75	2.74878	0.0709
FR_YIELD10YBY does not Granger Cause UKBRBASE_INDEX	75	2.61506	0.0803
IR_DEBTSEC_GDP does not Granger Cause DE_YIELD10YBY	74	2.4799	0.0912
IR_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	74	2.84746	0.0648
IR_DEBTSEC_GDP does not Granger Cause EURR002W_INDEX	74	4.07502	0.0212
IR_DEBTSEC_GDP does not Granger Cause FR_HUR	74	4.25663	0.0181
IR_DEBTSEC_GDP does not Granger Cause FR_YIELD10YBY	74	2.4024	0.0980
IR_DEBTSEC_GDP does not Granger Cause IR_HUR	74	5.65502	0.0053
IR_DEBTSEC_GDP does not Granger Cause NL_DEBTSEC_GDP	74	7.7526	0.0009
IR_DEBTSEC_GDP does not Granger Cause NL_YIELD10YBY	74	2.40413	0.0979
IR_DEBTSEC_GDP does not Granger Cause UK_DEBTSEC_GDP	74	8.46173	0.0005
IR_DEBTSEC_GDP does not Granger Cause UK_HUR	74	2.94965	0.0590
IR_DEBTSEC_GDP does not Granger Cause UKBRBASE_INDEX	74	2.94072	0.0595
IR_HUR does not Granger Cause DE_DEBTSEC_GDP	74	15.2189	0.0000
IR_HUR does not Granger Cause DK_DEBTSEC_GDP	74	7.11004	0.0016
IR_HUR does not Granger Cause DK_HUR	75	7.31205	0.0013
IR_HUR does not Granger Cause FR_DEBTSEC_GDP	74	4.60531	0.0133
IR_HUR does not Granger Cause FR_HUR	75	4.22355	0.0185
IR_HUR does not Granger Cause IR_DEBTSEC_GDP	74	5.90306	0.0043
IR_HUR does not Granger Cause NL_DEBTSEC_GDP	74	8.69457	0.0004
IR_HUR does not Granger Cause NL_HUR	75	2.40916	0.0973
IR_HUR does not Granger Cause UK_DEBTSEC_GDP	74	16.1126	0.0000
IR_HUR does not Granger Cause UK_HUR	74	4.87505	0.0105
IR_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	74	4.51147	0.0144
IR_YIELD10YBY does not Granger Cause DK_DEBTSEC_GDP	74	3.03917	0.0543
IR_YIELD10YBY does not Granger Cause EURR002W_INDEX	75	4.17193	0.0194
IR_YIELD10YBY does not Granger Cause NL_DEBTSEC_GDP	74	3.81456	0.0268
IR_YIELD10YBY does not Granger Cause NL_HUR	75	4.49591	0.0146
IR_YIELD10YBY does not Granger Cause UK_HUR	74	2.8709	0.0634
NL_DEBTSEC_GDP does not Granger Cause DE_DEBTSEC_GDP	74	6.52577	0.0025
NL_DEBTSEC_GDP does not Granger Cause DE_HUR	74	5.76463	0.0048
NL_DEBTSEC_GDP does not Granger Cause DK_HUR	74	6.2208	0.0033
NL_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	74	4.82993	0.0109
NL_DEBTSEC_GDP does not Granger Cause FR_HUR	74	20.8233	0.0000
NL_DEBTSEC_GDP does not Granger Cause IR_DEBTSEC_GDP	74	2.75326	0.0707
NL_DEBTSEC_GDP does not Granger Cause IR_HUR	74	10.2204	0.0001

Figure 7.27 - Pairwise Granger Causality tests between the SBY and Macroeconomic variables (2 lags) (2/4)

NL_DEBTSEC_GDP does not Granger Cause UK_HUR	74	4.92251	0.0100
NL_HUR does not Granger Cause DE_DEBTSEC_GDP	74	5.8456	0.0045
NL_HUR does not Granger Cause DK_DEBTSEC_GDP	74	3.43672	0.0378
NL_HUR does not Granger Cause DK_HUR	75	2.59326	0.0819
NL_HUR does not Granger Cause FR_HUR	75	2.7972	0.0678
NL_HUR does not Granger Cause IR_HUR	75	3.11316	0.0507
NL_HUR does not Granger Cause NL_DEBTSEC_GDP	74	4.72049	0.0120
NL_HUR does not Granger Cause UK_HUR	74	2.44088	0.0946
NL_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	74	3.37716	0.0399
NL_YIELD10YBY does not Granger Cause DE_HUR	75	5.37643	0.0067
NL_YIELD10YBY does not Granger Cause EURR002W_INDEX	75	5.00197	0.0093
NL_YIELD10YBY does not Granger Cause FR_HUR	75	2.58934	0.0822
NL_YIELD10YBY does not Granger Cause IR_HUR	75	3.07987	0.0522
NL_YIELD10YBY does not Granger Cause UKBRBASE_INDEX	75	2.66508	0.0767
UK_DEBTSEC_GDP does not Granger Cause DE_DEBTSEC_GDP	74	7.9186	0.0008
UK_DEBTSEC_GDP does not Granger Cause DE_HUR	74	5.30894	0.0072
UK_DEBTSEC_GDP does not Granger Cause DE_YIELD10YBY	74	5.10974	0.0085
UK_DEBTSEC_GDP does not Granger Cause DK_DEBTSEC_GDP	74	5.79157	0.0047
UK_DEBTSEC_GDP does not Granger Cause DK_HUR	74	10.3514	0.0001
UK_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	74	6.48331	0.0026
UK_DEBTSEC_GDP does not Granger Cause EURR002W_INDEX	74	2.79871	0.0678
UK_DEBTSEC_GDP does not Granger Cause FR_DEBTSEC_GDP	74	5.6908	0.0052
UK_DEBTSEC_GDP does not Granger Cause FR_HUR	74	3.93128	0.0242
UK_DEBTSEC_GDP does not Granger Cause FR_YIELD10YBY	74	3.43561	0.0378
UK_DEBTSEC_GDP does not Granger Cause IR_DEBTSEC_GDP	74	6.86238	0.0019
UK_DEBTSEC_GDP does not Granger Cause IR_HUR	74	7.39122	0.0012
UK_DEBTSEC_GDP does not Granger Cause NL_DEBTSEC_GDP	74	2.52844	0.0872
UK_DEBTSEC_GDP does not Granger Cause NL_YIELD10YBY	74	4.50576	0.0145
UK_DEBTSEC_GDP does not Granger Cause UK_YIELD10YBY	74	6.905	0.0018
UK_DEBTSEC_GDP does not Granger Cause UKBRBASE_INDEX	74	3.82306	0.0266
UK_HUR does not Granger Cause DE_DEBTSEC_GDP	74	9.78559	0.0002
UK_HUR does not Granger Cause DK_DEBTSEC_GDP	74	4.09045	0.0210
UK_HUR does not Granger Cause DK_HUR	74	6.52838	0.0025
UK_HUR does not Granger Cause FR_DEBTSEC_GDP	74	3.41033	0.0387
UK_HUR does not Granger Cause FR_HUR	74	2.41222	0.0971
UK_HUR does not Granger Cause IR_DEBTSEC_GDP	74	3.64496	0.0313
UK_HUR does not Granger Cause IR_HUR	74	7.48924	0.0011
UK_HUR does not Granger Cause NL_DEBTSEC_GDP	74	5.79092	0.0047
UK_HUR does not Granger Cause NL_HUR	74	2.57748	0.0833
UK_HUR does not Granger Cause UK_DEBTSEC_GDP	74	8.77533	0.0004
UK_YIELD10YBY does not Granger Cause DE_HUR	75	4.41627	0.0156
UK_YIELD10YBY does not Granger Cause EURR002W_INDEX	75	6.28376	0.0031
UK_YIELD10YBY does not Granger Cause FR_HUR	75	3.22442	0.0458
UK_YIELD10YBY does not Granger Cause IR_HUR	75	4.07839	0.0211
UK_YIELD10YBY does not Granger Cause UKBRBASE_INDEX	75	4.84246	0.0107

Figure 7.28 - Pairwise Granger Causality tests between the SBY and Macroeconomic variables (2 lags) (3/4)

UKBRBASE_INDEX does not Granger Cause DE_DEBTSEC_GDP	74	11.4657	0.0001
UKBRBASE_INDEX does not Granger Cause DE_HUR	75	5.83659	0.0045
UKBRBASE_INDEX does not Granger Cause DK_HUR	75	6.04689	0.0038
UKBRBASE_INDEX does not Granger Cause DK_YIELD10YBY	75	4.17058	0.0194
UKBRBASE_INDEX does not Granger Cause EURR002W_INDEX	75	3.04653	0.0539
UKBRBASE_INDEX does not Granger Cause FR_DEBTSEC_GDP	74	12.9493	0.0000
UKBRBASE_INDEX does not Granger Cause FR_HUR	75	10.9731	0.0001
UKBRBASE_INDEX does not Granger Cause IR_DEBTSEC_GDP	74	4.2919	0.0175
UKBRBASE_INDEX does not Granger Cause IR_HUR	75	11.4201	0.0001
UKBRBASE_INDEX does not Granger Cause NL_YIELD10YBY	75	3.31289	0.0422
UKBRBASE_INDEX does not Granger Cause UK_DEBTSEC_GDP	74	6.74666	0.0021
UKBRBASE_INDEX does not Granger Cause UK_HUR	74	9.19128	0.0003
UKBRBASE_INDEX does not Granger Cause UK_YIELD10YBY	75	2.48023	0.0911

Figure 7.29 - Pairwise Granger Causality tests between the SBY and Macroeconomic variables (2 lags) (4/4)

Pairwise Granger Causality Tests macroeconomic variables (Debt securities as % of GDP, Harmonized Unemployment Rate and the Reference rate) and all SBY and 4 lags (excluding Granger Causality between the SBY).

Null Hypothesis:	Obs	F-Statistic	Prob.
DE_DEBTSEC_GDP does not Granger Cause DE_HUR	72	2.37961	0.0610
DE_DEBTSEC_GDP does not Granger Cause DE_YIELD10YBY	72	2.73478	0.0365
DE_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	72	2.56637	0.0466
DE_DEBTSEC_GDP does not Granger Cause NL_YIELD10YBY	72	2.59095	0.0450
DE_DEBTSEC_GDP does not Granger Cause UK_YIELD10YBY	72	2.35067	0.0636
DE_HUR does not Granger Cause DE_DEBTSEC_GDP	72	3.66869	0.0095
DE_HUR does not Granger Cause EURR002W_INDEX	73	3.30943	0.0158
DE_HUR does not Granger Cause IR_HUR	73	3.57116	0.0108
DE_HUR does not Granger Cause UKBRBASE_INDEX	73	2.30576	0.0677
DE_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	72	3.68153	0.0093
DE_YIELD10YBY does not Granger Cause DE_HUR	73	3.22761	0.0178
DE_YIELD10YBY does not Granger Cause EURR002W_INDEX	73	5.20209	0.0011
DE_YIELD10YBY does not Granger Cause FR_HUR	73	3.72139	0.0087
DE_YIELD10YBY does not Granger Cause IR_HUR	73	2.11198	0.0895
DE_YIELD10YBY does not Granger Cause UK_HUR	72	2.45697	0.0546
DE_YIELD10YBY does not Granger Cause UKBRBASE_INDEX	73	3.55834	0.0110
DK_DEBTSEC_GDP does not Granger Cause DK_HUR	72	2.20544	0.0785
DK_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	72	3.61906	0.0102
DK_DEBTSEC_GDP does not Granger Cause FR_DEBTSEC_GDP	72	4.01804	0.0058
DK_DEBTSEC_GDP does not Granger Cause FR_HUR	72	2.13738	0.0865
DK_DEBTSEC_GDP does not Granger Cause UK_DEBTSEC_GDP	72	3.69591	0.0091
DK_HUR does not Granger Cause DK_DEBTSEC_GDP	72	2.95737	0.0265
DK_HUR does not Granger Cause FR_DEBTSEC_GDP	72	7.19736	0.0001
DK_HUR does not Granger Cause IR_HUR	73	3.67832	0.0093
DK_HUR does not Granger Cause UK_DEBTSEC_GDP	72	3.57774	0.0108
DK_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	72	2.24354	0.0743
DK_YIELD10YBY does not Granger Cause DE_HUR	73	3.29081	0.0162
DK_YIELD10YBY does not Granger Cause IR_HUR	73	2.13082	0.0871
EURR002W_INDEX does not Granger Cause DE_DEBTSEC_GDP	72	13.9529	0.0000
EURR002W_INDEX does not Granger Cause DE_HUR	73	4.1498	0.0047
EURR002W_INDEX does not Granger Cause DK_YIELD10YBY	73	4.05315	0.0054
EURR002W_INDEX does not Granger Cause FR_DEBTSEC_GDP	72	5.67953	0.0006
EURR002W_INDEX does not Granger Cause FR_HUR	73	2.26312	0.0720
EURR002W_INDEX does not Granger Cause IR_HUR	73	3.59004	0.0106
FR_DEBTSEC_GDP does not Granger Cause DE_DEBTSEC_GDP	72	3.84113	0.0074
FR_DEBTSEC_GDP does not Granger Cause DE_HUR	72	3.81177	0.0077
FR_DEBTSEC_GDP does not Granger Cause DE_YIELD10YBY	72	2.24913	0.0737
FR_DEBTSEC_GDP does not Granger Cause DK_DEBTSEC_GDP	72	2.71824	0.0374
FR_DEBTSEC_GDP does not Granger Cause DK_HUR	72	2.88461	0.0294
FR_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	72	3.83376	0.0075
FR_DEBTSEC_GDP does not Granger Cause NL_YIELD10YBY	72	3.07177	0.0224
FR_DEBTSEC_GDP does not Granger Cause UK_DEBTSEC_GDP	72	3.95096	0.0063
FR_DEBTSEC_GDP does not Granger Cause UK_YIELD10YBY	72	2.75767	0.0353
FR_HUR does not Granger Cause DE_DEBTSEC_GDP	72	11.6299	0.0000
FR_HUR does not Granger Cause FR_DEBTSEC_GDP	72	3.27977	0.0166

Figure 7.30 - Pairwise Granger Causality tests between the SBY and Macroeconomic variables (4 lags) (1/3)

FR_HUR does not Granger Cause IR_DEBTSEC_GDP	72	2.62185	0.0430
FR_HUR does not Granger Cause IR_HUR	73	5.43899	0.0008
FR_HUR does not Granger Cause NL_HUR	73	2.67196	0.0398
FR_HUR does not Granger Cause UK_DEBTSEC_GDP	72	5.44378	0.0008
FR_HUR does not Granger Cause UK_HUR	72	5.43209	0.0008
FR_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	72	2.61845	0.0432
FR_YIELD10YBY does not Granger Cause DE_HUR	73	2.92539	0.0276
FR_YIELD10YBY does not Granger Cause EURR002W_INDEX	73	2.67054	0.0399
FR_YIELD10YBY does not Granger Cause FR_HUR	73	3.59608	0.0105
FR_YIELD10YBY does not Granger Cause UK_HUR	72	2.52217	0.0497
FR_YIELD10YBY does not Granger Cause UKBRBASE_INDEX	73	2.33127	0.0653
IR_DEBTSEC_GDP does not Granger Cause DE_YIELD10YBY	72	2.50588	0.0509
IR_DEBTSEC_GDP does not Granger Cause DK_HUR	72	2.41801	0.0577
IR_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	72	2.10551	0.0906
IR_DEBTSEC_GDP does not Granger Cause EURR002W_INDEX	72	2.29471	0.0690
IR_DEBTSEC_GDP does not Granger Cause FR_HUR	72	3.13008	0.0206
IR_DEBTSEC_GDP does not Granger Cause FR_YIELD10YBY	72	2.05394	0.0975
IR_DEBTSEC_GDP does not Granger Cause NL_DEBTSEC_GDP	72	2.74106	0.0362
IR_DEBTSEC_GDP does not Granger Cause NL_YIELD10YBY	72	2.46594	0.0539
IR_DEBTSEC_GDP does not Granger Cause UK_DEBTSEC_GDP	72	2.84738	0.0310
IR_DEBTSEC_GDP does not Granger Cause UK_YIELD10YBY	72	2.54325	0.0482
IR_HUR does not Granger Cause DE_DEBTSEC_GDP	72	7.47216	0.0001
IR_HUR does not Granger Cause DK_DEBTSEC_GDP	72	4.33968	0.0037
IR_HUR does not Granger Cause DK_HUR	73	3.52995	0.0115
IR_HUR does not Granger Cause DK_YIELD10YBY	73	2.3792	0.0609
IR_HUR does not Granger Cause EURR002W_INDEX	73	2.55645	0.0471
IR_HUR does not Granger Cause FR_DEBTSEC_GDP	72	4.34768	0.0036
IR_HUR does not Granger Cause FR_HUR	73	2.191	0.0799
IR_HUR does not Granger Cause IR_DEBTSEC_GDP	72	2.04185	0.0992
IR_HUR does not Granger Cause NL_DEBTSEC_GDP	72	4.06222	0.0054
IR_HUR does not Granger Cause UK_DEBTSEC_GDP	72	4.47243	0.0030
IR_HUR does not Granger Cause UK_HUR	72	2.4833	0.0525
IR_HUR does not Granger Cause UKBRBASE_INDEX	73	2.04875	0.0980
IR_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	72	3.05109	0.0231
IR_YIELD10YBY does not Granger Cause EURR002W_INDEX	73	2.57394	0.0459
IR_YIELD10YBY does not Granger Cause NL_HUR	73	2.41564	0.0578
IR_YIELD10YBY does not Granger Cause UK_HUR	72	2.4601	0.0543
NL_DEBTSEC_GDP does not Granger Cause DE_DEBTSEC_GDP	72	11.415	0.0000
NL_DEBTSEC_GDP does not Granger Cause DE_HUR	72	2.19308	0.0799
NL_DEBTSEC_GDP does not Granger Cause DK_HUR	72	3.9449	0.0064
NL_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	72	2.68551	0.0392
NL_DEBTSEC_GDP does not Granger Cause FR_DEBTSEC_GDP	72	3.68183	0.0093
NL_DEBTSEC_GDP does not Granger Cause FR_HUR	72	10.5834	0.0000
NL_DEBTSEC_GDP does not Granger Cause IR_HUR	72	2.87601	0.0298
NL_DEBTSEC_GDP does not Granger Cause NL_HUR	72	3.7587	0.0083
NL_DEBTSEC_GDP does not Granger Cause UK_HUR	72	5.00956	0.0014
NL_HUR does not Granger Cause DK_DEBTSEC_GDP	72	2.23681	0.0750
NL_HUR does not Granger Cause FR_DEBTSEC_GDP	72	2.47393	0.0533
NL_HUR does not Granger Cause NL_DEBTSEC_GDP	72	2.39744	0.0595
NL_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	72	2.93585	0.0273
NL_YIELD10YBY does not Granger Cause DE_HUR	73	3.03218	0.0236
NL_YIELD10YBY does not Granger Cause EURR002W_INDEX	73	3.6425	0.0098
NL_YIELD10YBY does not Granger Cause FR_DEBTSEC_GDP	72	2.16449	0.0832
NL_YIELD10YBY does not Granger Cause FR_HUR	73	2.86786	0.0300
NL_YIELD10YBY does not Granger Cause UKBRBASE_INDEX	73	3.08231	0.0220

Figure 7.31 - Pairwise Granger Causality tests between the SBY and Macroeconomic variables (4 lags) (2/3)

UK_DEBTSEC_GDP does not Granger Cause DE_DEBTSEC_GDP	72	2.52401	0.0495
UK_DEBTSEC_GDP does not Granger Cause DE_HUR	72	3.02835	0.0239
UK_DEBTSEC_GDP does not Granger Cause DE_YIELD10YBY	72	5.39741	0.0008
UK_DEBTSEC_GDP does not Granger Cause DK_DEBTSEC_GDP	72	2.99006	0.0252
UK_DEBTSEC_GDP does not Granger Cause DK_HUR	72	3.92536	0.0066
UK_DEBTSEC_GDP does not Granger Cause DK_YIELD10YBY	72	4.98948	0.0015
UK_DEBTSEC_GDP does not Granger Cause EURR002W_INDEX	72	3.73892	0.0086
UK_DEBTSEC_GDP does not Granger Cause FR_DEBTSEC_GDP	72	6.82052	0.0001
UK_DEBTSEC_GDP does not Granger Cause FR_YIELD10YBY	72	3.24905	0.0174
UK_DEBTSEC_GDP does not Granger Cause IR_DEBTSEC_GDP	72	2.78297	0.0341
UK_DEBTSEC_GDP does not Granger Cause IR_YIELD10YBY	72	7.21792	0.0001
UK_DEBTSEC_GDP does not Granger Cause NL_YIELD10YBY	72	4.68408	0.0023
UK_DEBTSEC_GDP does not Granger Cause UK_YIELD10YBY	72	6.35465	0.0002
UK_DEBTSEC_GDP does not Granger Cause UKBRBASE_INDEX	72	3.39299	0.0141
UK_HUR does not Granger Cause DE_DEBTSEC_GDP	72	3.78553	0.0080
UK_HUR does not Granger Cause DK_HUR	72	2.47672	0.0530
UK_HUR does not Granger Cause FR_DEBTSEC_GDP	72	2.2699	0.0715
UK_HUR does not Granger Cause FR_HUR	72	2.20498	0.0785
UK_HUR does not Granger Cause NL_DEBTSEC_GDP	72	2.64358	0.0417
UK_HUR does not Granger Cause UK_DEBTSEC_GDP	72	2.08739	0.0929
UK_YIELD10YBY does not Granger Cause DE_DEBTSEC_GDP	72	4.46993	0.0030
UK_YIELD10YBY does not Granger Cause DE_HUR	73	2.35242	0.0633
UK_YIELD10YBY does not Granger Cause EURR002W_INDEX	73	5.27928	0.0010
UK_YIELD10YBY does not Granger Cause FR_HUR	73	2.993	0.0250
UK_YIELD10YBY does not Granger Cause IR_HUR	73	2.29553	0.0687
UK_YIELD10YBY does not Granger Cause UK_HUR	72	3.03181	0.0238
UK_YIELD10YBY does not Granger Cause UKBRBASE_INDEX	73	4.13338	0.0048
UKBRBASE_INDEX does not Granger Cause DE_DEBTSEC_GDP	72	16.1857	0.0000
UKBRBASE_INDEX does not Granger Cause DE_HUR	73	2.84702	0.0309
UKBRBASE_INDEX does not Granger Cause DK_HUR	73	3.72868	0.0086
UKBRBASE_INDEX does not Granger Cause DK_YIELD10YBY	73	4.44724	0.0031
UKBRBASE_INDEX does not Granger Cause FR_DEBTSEC_GDP	72	8.97027	0.0000
UKBRBASE_INDEX does not Granger Cause FR_HUR	73	5.27813	0.0010
UKBRBASE_INDEX does not Granger Cause IR_HUR	73	6.27496	0.0003
UKBRBASE_INDEX does not Granger Cause UK_HUR	72	4.81053	0.0019

Figure 7.32 - Pairwise Granger Causality tests between the SBY and Macroeconomic variables (4 lags) (3/3)

7.5 Regression outputs

Dependent Variable: DE_YIELD10YBY
Method: Least Squares
Date: 09/14/19 Time: 12:11
Sample (adjusted): 1 76
Included observations: 76 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.602606	0.141550	4.257207	0.0001
DE_DEBTSEC_GDP	-0.928034	0.388056	-2.391492	0.0195
UK_YIELD10YBY	0.302734	0.026750	11.31700	0.0000
IR_YIELD10YBY	0.018435	0.007828	2.354896	0.0214
NL_YIELD10YBY	0.676678	0.029108	23.24736	0.0000
NL_DEBTSEC_GDP	-1.558347	0.404167	-3.855701	0.0003
DK_DEBTSEC_GDP	0.541952	0.220190	2.461295	0.0163

R-squared	0.998583	Mean dependent var	2.749053
Adjusted R-squared	0.998459	S.D. dependent var	1.683819
S.E. of regression	0.066092	Akaike info criterion	-2.507960
Sum squared resid	0.301401	Schwarz criterion	-2.293287
Log likelihood	102.3025	Hannan-Quinn criter.	-2.422166
F-statistic	8101.948	Durbin-Watson stat	1.255072
Prob(F-statistic)	0.000000		

Figure 7.33 - Main output of the Equation A

Figure 7.34 - Residual and Actual vs Fitted graph of the Equation A

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 2 lags

F-statistic	5.185641	Prob. F(2,67)	0.0081
Obs*R-squared	10.18747	Prob. Chi-Square(2)	0.0061

Figure 7.35 - BG Serial Correlation test of the Equation A

Heteroskedasticity Test: Breusch-Pagan-Godfrey
Null hypothesis: Homoskedasticity

F-statistic	1.696956	Prob. F(6,69)	0.1348
Obs*R-squared	9.772604	Prob. Chi-Square(6)	0.1346
Scaled explained SS	11.43029	Prob. Chi-Square(6)	0.0760

Figure 7.36 - BPG Heteroskedasticity test of the Equation A

Dependent Variable: UK_YIELD10YBY
Method: Least Squares
Date: 01/02/20 Time: 02:04
Sample (adjusted): 1 76
Included observations: 76 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.473690	0.604812	-5.743425	0.0000
DE_YIELD10YBY	1.369232	0.125464	10.91337	0.0000
FR_YIELD10YBY	-0.327960	0.152653	-2.148394	0.0352
DE_DEBTSEC_GDP	10.99003	1.203918	9.128553	0.0000
DK_DEBTSEC_GDP	1.410113	0.369603	3.815208	0.0003
IR_DEBTSEC_GDP	1.157649	0.384163	3.013431	0.0036
IR_HUR	-8.456701	3.458190	-2.445413	0.0171
UK_HUR	-22.80388	8.997496	-2.534469	0.0136

R-squared	0.988307	Mean dependent var	3.391053
Adjusted R-squared	0.987103	S.D. dependent var	1.450772
S.E. of regression	0.164754	Akaike info criterion	-0.669427
Sum squared resid	1.845782	Schwarz criterion	-0.424087
Log likelihood	33.43823	Hannan-Quinn criter.	-0.571377
F-statistic	821.0734	Durbin-Watson stat	1.203203
Prob(F-statistic)	0.000000		

Figure 7.37 - Main output of the Equation B

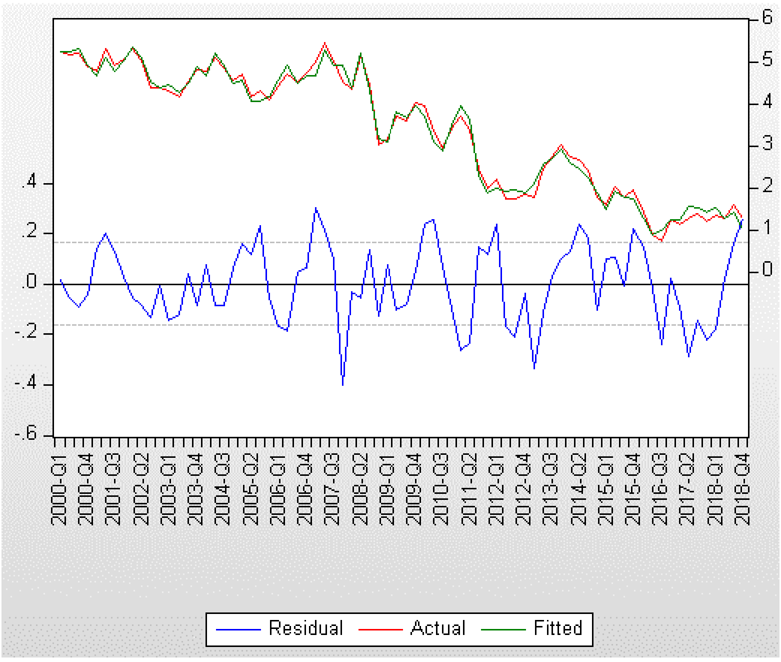


Figure 7.38 - Residual and Actual vs Fitted graph of the Equation B

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	6.681778	Prob. F(2,66)	0.0023
Obs*R-squared	12.79719	Prob. Chi-Square(2)	0.0017

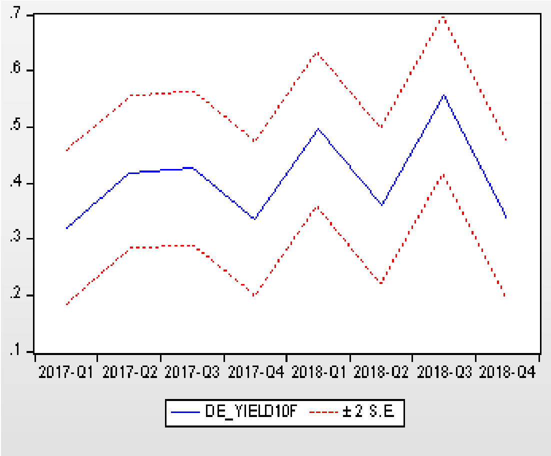
Figure 7.39 - BG Serial Correlation test of the Equation B

Heteroskedasticity Test: Breusch-Pagan-Godfrey

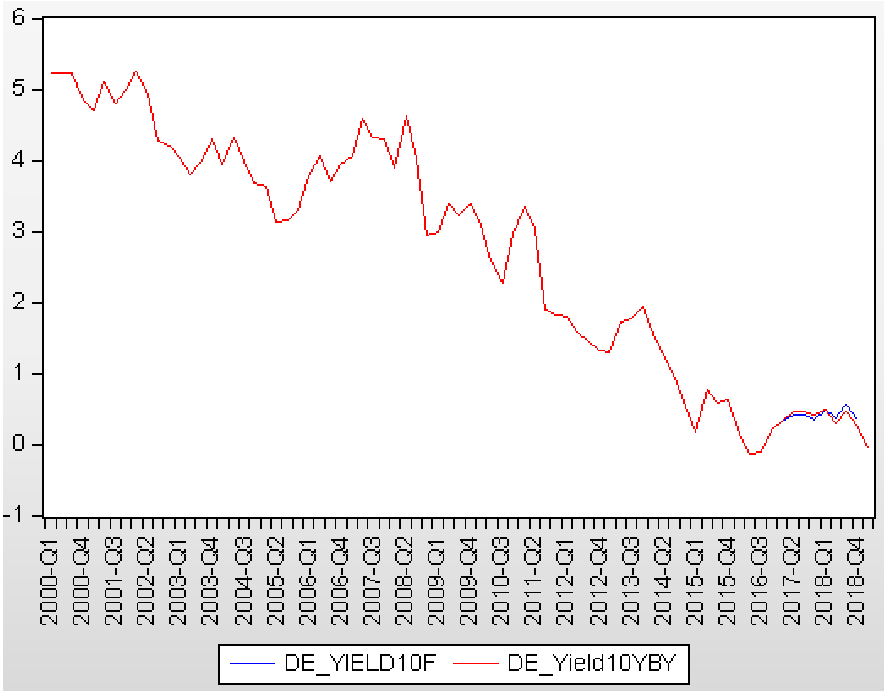
F-statistic	1.095045	Prob. F(7,68)	0.3763
Obs*R-squared	7.699221	Prob. Chi-Square(7)	0.3599
Scaled explained SS	4.418359	Prob. Chi-Square(7)	0.7305

Figure 7.40 - BPG Heteroskedasticity test of the Equation B

7.6 Forecasting

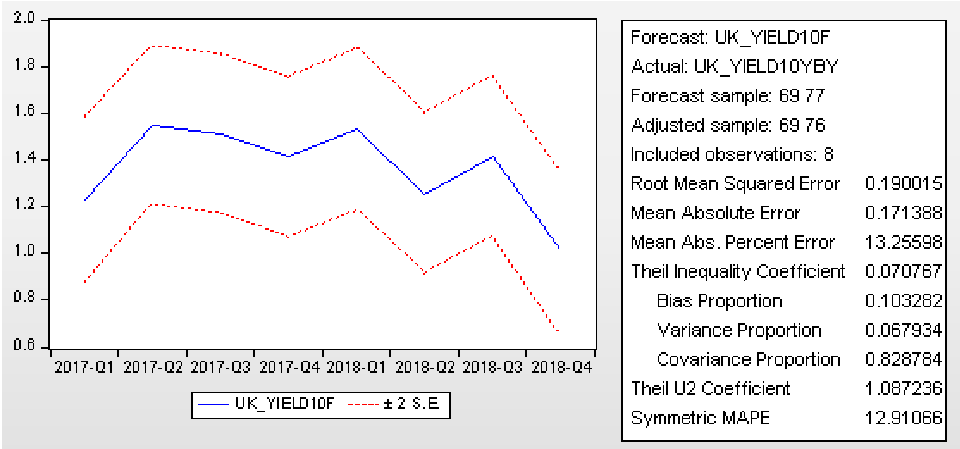


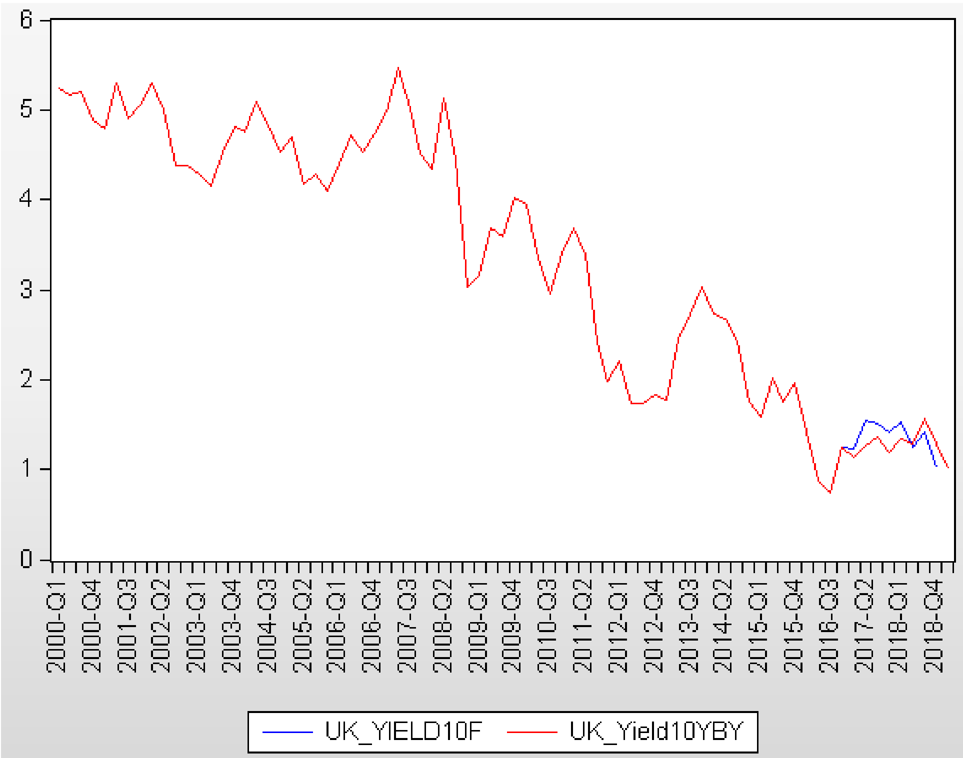
Forecast: DE_YIELD10F	
Actual: DE_YIELD10YBY	
Forecast sample: 69 77	
Adjusted sample: 69 76	
Included observations: 8	
Root Mean Squared Error	0.063338
Mean Absolute Error	0.053358
Mean Abs. Percent Error	14.96548
Theil Inequality Coefficient	0.076968
Bias Proportion	0.010491
Variance Proportion	0.023537
Covariance Proportion	0.965971
Theil U2 Coefficient	0.471411
Symmetric MAPE	14.20436



	DE_YIELD10F	DE_YIELD10YBY
2015-Q2	0.764000	0.764
2015-Q3	0.587000	0.587
2015-Q4	0.629000	0.629
2016-Q1	0.153000	0.153
2016-Q2	-0.130000	-0.130
2016-Q3	-0.119000	-0.119
2016-Q4	0.208000	0.208
2017-Q1	0.319991	0.328
2017-Q2	0.418118	0.466
2017-Q3	0.426026	0.464
2017-Q4	0.334514	0.427
2018-Q1	0.495870	0.497
2018-Q2	0.361859	0.302
2018-Q3	0.555145	0.470
2018-Q4	0.336379	0.242
2019-Q1	NA	-0.070

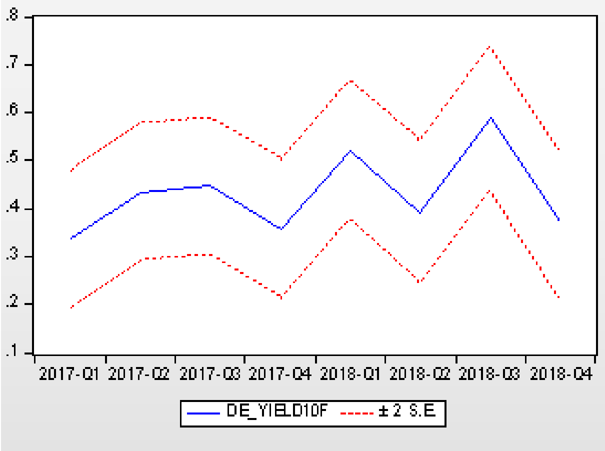
Figure 7.41 - In-sample Forecast outputs for model A



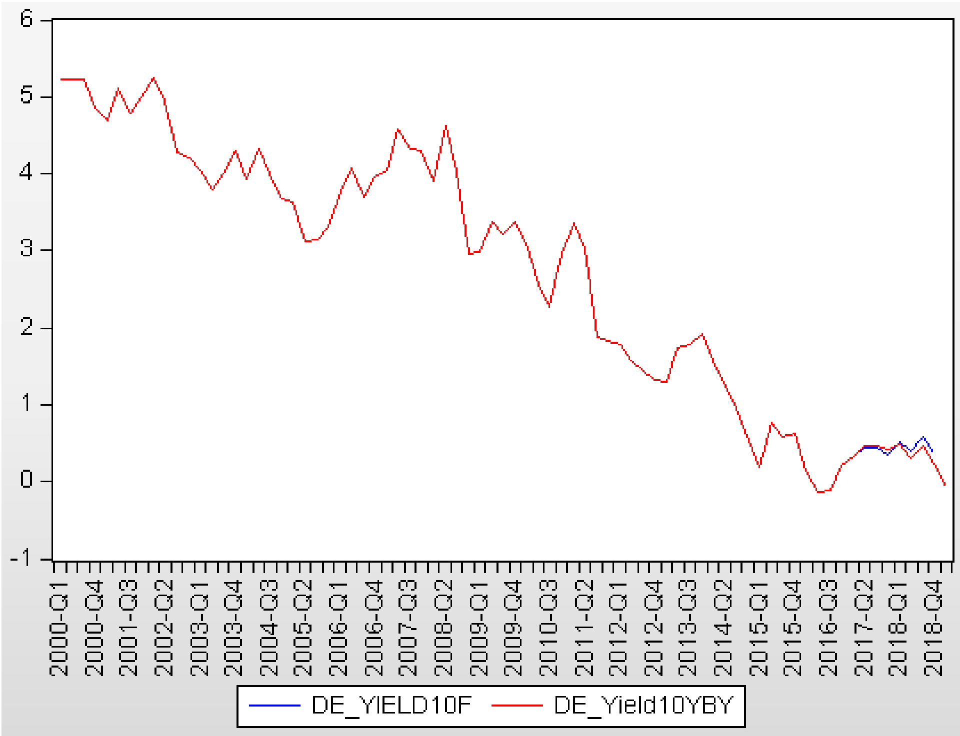


	UK_YIELD10F	UK_YIELD10YBY
2015-Q3	1.762000	1.762
2015-Q4	1.960000	1.960
2016-Q1	1.415000	1.415
2016-Q2	0.867000	0.867
2016-Q3	0.746000	0.746
2016-Q4	1.239000	1.239
2017-Q1	1.230945	1.139
2017-Q2	1.545557	1.257
2017-Q3	1.511113	1.365
2017-Q4	1.410584	1.190
2018-Q1	1.532617	1.350
2018-Q2	1.255380	1.278
2018-Q3	1.416681	1.573
2018-Q4	1.014654	1.277
2019-Q1	NA	1.000

Figure 7.42 - In-sample Forecast outputs for model B

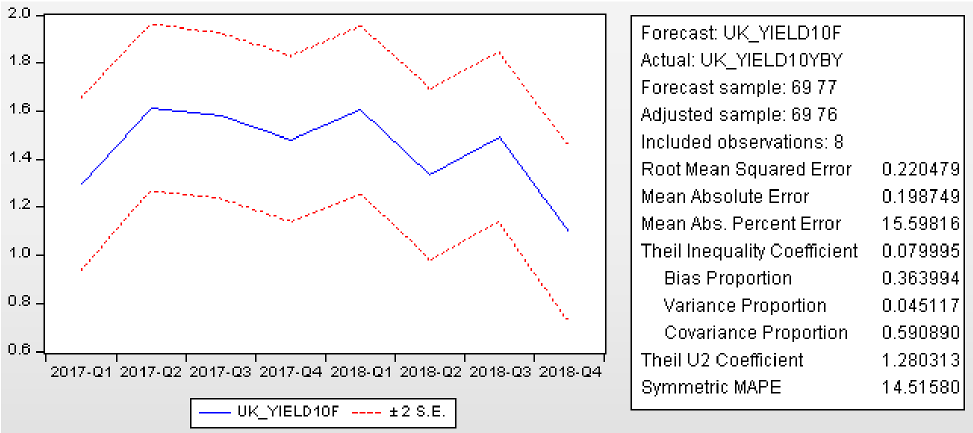


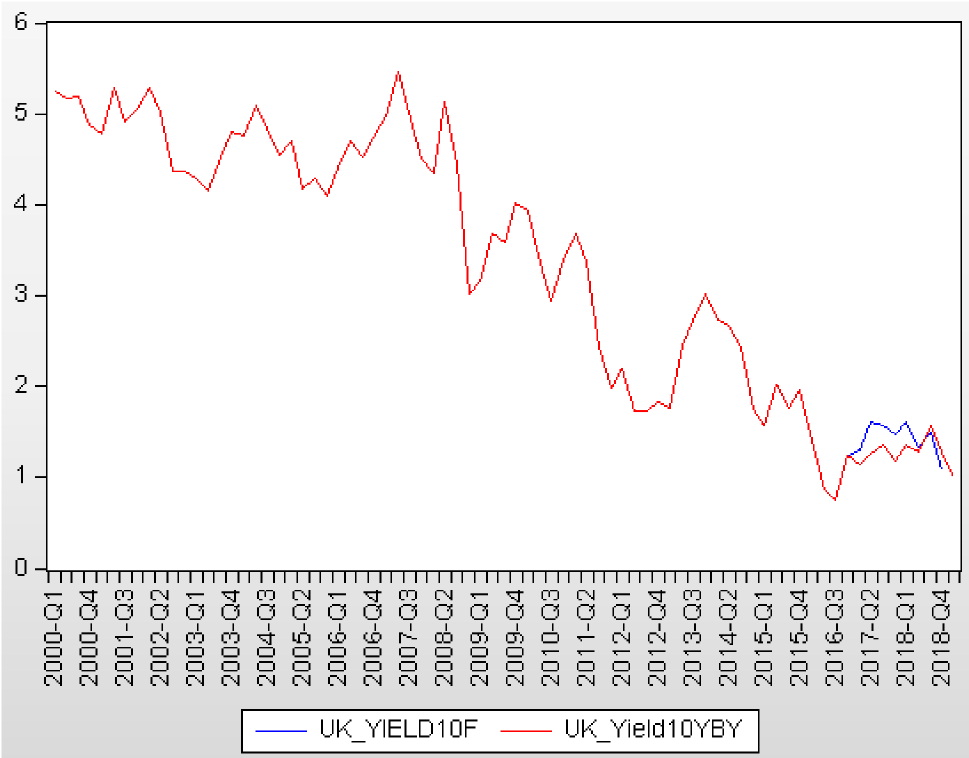
Forecast: DE_YIELD10F	
Actual: DE_YIELD10YBY	
Forecast sample: 69 77	
Adjusted sample: 69 76	
Included observations: 8	
Root Mean Squared Error	0.075404
Mean Absolute Error	0.060883
Mean Abs. Percent Error	17.78647
Theil Inequality Coefficient	0.088996
Bias Proportion	0.168627
Variance Proportion	0.011188
Covariance Proportion	0.820184
Theil U2 Coefficient	0.570750
Symmetric MAPE	15.75935



	DE_YIELD10F	DE_YIELD10YBY
2015-Q2	0.764000	0.764
2015-Q3	0.587000	0.587
2015-Q4	0.629000	0.629
2016-Q1	0.153000	0.153
2016-Q2	-0.130000	-0.130
2016-Q3	-0.119000	-0.119
2016-Q4	0.208000	0.208
2017-Q1	0.335898	0.328
2017-Q2	0.434347	0.466
2017-Q3	0.447113	0.464
2017-Q4	0.355865	0.427
2018-Q1	0.520243	0.497
2018-Q2	0.391410	0.302
2018-Q3	0.587178	0.470
2018-Q4	0.371660	0.242
2019-Q1	NA	-0.070

Figure 7.43 - Out-of-sample Forecast outputs for model A





	UK_YIELD10F	UK_YIELD10YBY
2015-Q3	1.762000	1.762
2015-Q4	1.960000	1.960
2016-Q1	1.415000	1.415
2016-Q2	0.867000	0.867
2016-Q3	0.746000	0.746
2016-Q4	1.239000	1.239
2017-Q1	1.298150	1.139
2017-Q2	1.613638	1.257
2017-Q3	1.578099	1.365
2017-Q4	1.478185	1.190
2018-Q1	1.604720	1.350
2018-Q2	1.333281	1.278
2018-Q3	1.493524	1.573
2018-Q4	1.093556	1.277
2019-Q1	NA	1.000

Figure 7.44 - Out-of-sample Forecast outputs for model B